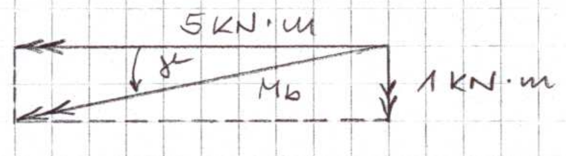
Aufgabe 1



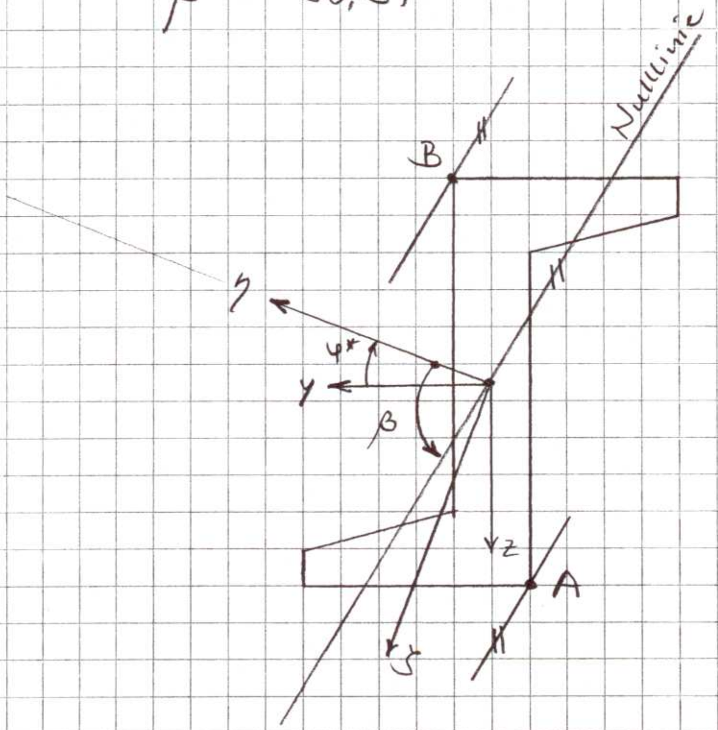
$$M_b = \sqrt{(5\,kN\cdot m)^2 + (1\,kN\cdot m)^2} = 5{,}10\,kN\cdot m$$

$$\gamma = \arctan\left(\frac{1\,kN\cdot m}{5\,kN\cdot m}\right) = 11{,}31^\circ$$ (1)

$$\alpha = -\varphi^* + \gamma = 20{,}11^\circ + 11{,}31^\circ = 31{,}42^\circ$$

$$\tan\beta = \frac{550\,cm^4}{57\,cm^4}\cdot \tan 31{,}42^\circ = 5{,}894$$

$$\beta = 80{,}37^\circ$$ (1)



(1)

2) $$M_{b\eta} = M_b \cdot \cos\alpha = 5{,}10\,kN\cdot m \cdot \cos 31{,}42^\circ = 4{,}35\,kN\cdot m$$ (1)

$$M_{b\zeta} = M_b \cdot \sin\alpha = 5{,}10\,kN\cdot m \cdot \sin 31{,}42^\circ = 2{,}66\,kN\cdot m$$

Punkt A

$y_A = -1\,cm$; $z_A = 5{,}5\,cm$

$$\eta_A = 5{,}5\,cm \cdot \sin(-20{,}11^\circ) + (-1\,cm) \cdot \cos(-20{,}11^\circ)$$

$$\eta_A = -2{,}83\,cm$$

$$\zeta_A = 5{,}5\,cm \cdot \cos(-20{,}11^\circ) - (-1\,cm) \cdot \sin(-20{,}11^\circ)$$

$$\zeta_A = 4{,}82\,cm$$ 1

Punkt B

$$y_B = 1\,cm \; ; \; z_B = -5{,}5\,cm$$

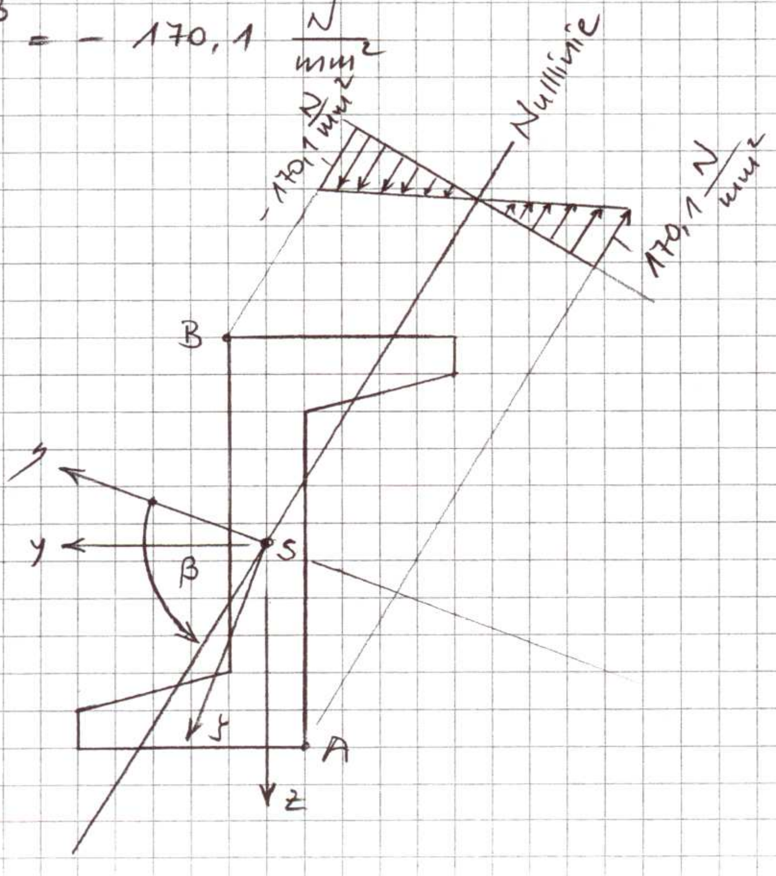
$$\eta_B = 2{,}83\,cm \; ; \; \zeta_B = -4{,}82\,cm$$ 1

$$\sigma_b^A = \frac{435\,kN \cdot cm}{550\,cm^4} \cdot 4{,}82\,cm - \frac{266\,kN \cdot cm}{57\,cm^4} \cdot (-2{,}83\,cm)$$

$$\sigma_b^A = 17{,}01\,\frac{kN}{cm^2} = 170{,}1\,\frac{N}{mm^2}$$ 1

$$\sigma_b^B = -170{,}1\,\frac{N}{mm^2}$$ 1

3/



2

10 P

$S_{y1} = 0$

$S_{y2} = 6\,cm \cdot 3\,cm \cdot 6{,}83\,cm = 122{,}94\,cm^3$

$S_{y3} = 122{,}94\,cm^3 + 2 \cdot 2\,cm \cdot 6\,cm \cdot 2{,}33\,cm = 178{,}86\,cm^3$

$S_{y4} = 178{,}86\,cm^3 - 2 \cdot 6\,cm \cdot 1{,}67\,cm - 2 \cdot \frac{2\,cm \cdot 2\,cm}{2} \cdot 2{,}00\,cm$

$S_{y4} = 150{,}82\,cm^3$ ③

$S_{y5} = 150{,}82\,cm^3 - 2\,cm \cdot 10\,cm \cdot 3{,}67\,cm = 77{,}42\,cm^3$

$S_{y6} = 77{,}42\,cm^3 - 4\,cm \cdot 2\,cm \cdot 5{,}67\,cm - 2 \cdot \frac{2\,cm \cdot 3\,cm}{2} \cdot 5{,}34\,cm$

$S_{y6} = 0{,}02 \approx 0$

Schnitt	S_y cm^3	b cm	τ kN/cm^2	τ N/mm^2
1	0	6	0	0
2	122,94	unten 6	1,691	16,91
		oben 4	2,537	25,37
3	178,86	unten 4	3,689	36,89
		oben 6	2,459	24,59
4	150,82	10	1,244	12,44
5	77,42	10	0,639	6,39
6	0	4	0	

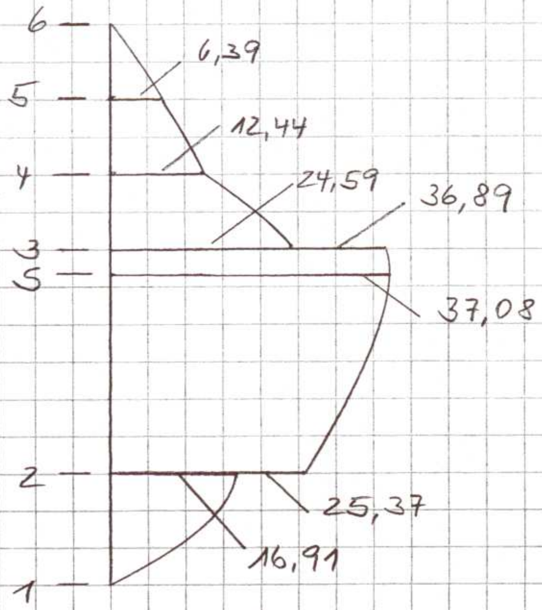
②

2) $S_{ymax} = 122{,}94\,cm^3 + 2 \cdot 2 \cdot \frac{(5{,}33\,cm)^2}{2} = 179{,}76\,cm^3$

$\tau_{max} = \frac{150\,kN \cdot 179{,}76\,cm^3}{1818\,cm^4 \cdot 4\,cm} = 3{,}708\,\frac{kN}{cm^2} = 37{,}08\,\frac{N}{mm^2}$

Tritt in der Faser $z = 0$ auf ①

3)



τ in N/mm^2

②

$$\tau_{qzul} = \frac{70\,\frac{N}{mm^2}}{37{,}08\,\frac{N}{mm^2}} \cdot 150\,kN = 283{,}2\,kN$$ ①

9P

1) $I_y = I_z = \dfrac{a^4}{12} = \dfrac{(10\,cm)^4}{12} = 833{,}\bar{3}\,cm^4$

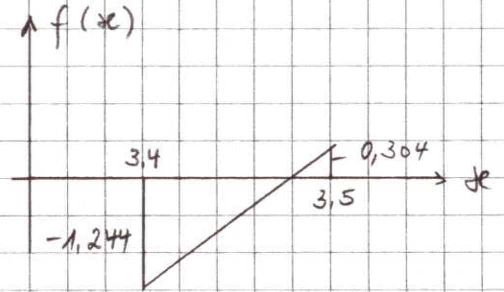
$$\varkappa = \frac{8 \cdot 10^8\,N \cdot mm \cdot 3000\,mm}{2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot 833{,}\bar{3} \cdot 10^4\,mm^4} = 1{,}371$$

Knickbedingung:

$$f(\varkappa) = (1{,}371 + \varkappa^2) \cdot \tan\varkappa - 1{,}371 \cdot \varkappa = 0$$

$f(3{,}4) = -1{,}244$

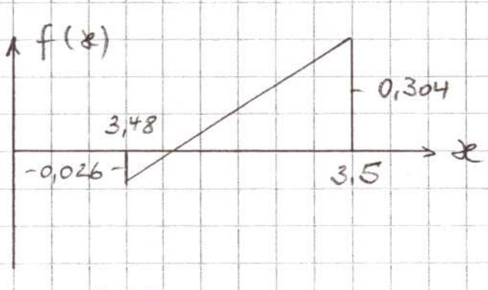
$f(3{,}5) = 0{,}304$



$$\frac{\Delta\varkappa}{1{,}244} = \frac{0{,}1}{1{,}244 + 0{,}304} \Rightarrow \Delta\varkappa = 0{,}080$$

$\varkappa_{Neu} = 3{,}480$

$f(\varkappa_{Neu}) = -0{,}026$



$$\Delta\varkappa = 0{,}026 \cdot \frac{0{,}02}{0{,}026 + 0{,}304} = 0{,}002$$

$\varkappa_{Neu} = 3{,}482$

$f(\varkappa_{Neu}) = 0{,}006 \sim 0$

$$\varkappa = \sqrt{\frac{F_K}{E \cdot I}} \cdot l$$

$$F_K = \frac{\varkappa^2 \cdot E \cdot I}{l^2} = \frac{3{,}482^2 \cdot 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot 833{,}\bar{3} \cdot 10^4\,mm^4}{(3000\,mm)^2}$$

$$F_K = 2357507\,N$$

$$F = 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot (100\,mm)^2 \cdot 1{,}2 \cdot 10^{-5}\,K^{-1} \cdot 30\,K$$

$$F = 756\,000\,N$$

$$\nu_K = \frac{2357507\,N}{756\,000\,N} = 3{,}12$$

2) $$2357507 = \frac{E \cdot I \cdot \pi^2}{s_K^2}$$

$$s_K = \sqrt{\frac{2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot 833{,}\bar{3} \cdot 10^4\,mm^4 \cdot \pi^2}{2357507}} = 2707\,mm$$

3) $$i = \sqrt{\frac{I}{A}} = \frac{1}{\sqrt{12}} \cdot a = 28{,}86\,mm$$

$$\lambda = \frac{s_K}{i} = \frac{2707\,mm}{28{,}86\,mm} = 93{,}8 > 85 = \lambda_{grenz}$$

Elastische Rechnung o.k.

4) $$\sigma = 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot \left(\frac{0{,}6\,mm}{3000\,mm} - 1{,}2 \cdot 10^{-5}\,K^{-1} \cdot 30\,K\right)$$

$$\sigma = -33{,}6\,\frac{N}{mm^2}$$

$$F = 33{,}6\,\frac{N}{mm^2} \cdot (100\,mm)^2 = 336\,000\,N$$

$$\nu = \frac{2357507\,N}{336\,000\,N} = 7{,}02$$