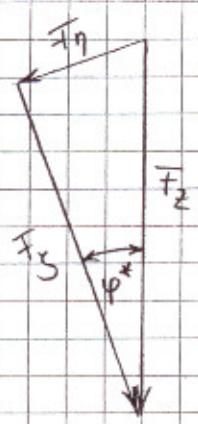
1)



$$F_\eta = F_z \cdot \sin\varphi^* = 20\,kN \cdot \sin 18{,}43^\circ = 6{,}32\,kN$$

$$F_\zeta = F_z \cdot \cos\varphi^* = 20\,kN \cdot \cos 18{,}43^\circ = 18{,}97\,kN$$

$$f_\eta = \frac{6320\,N \cdot (4000\,mm)^3}{192 \cdot 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot \frac{25}{30} \cdot a^4} = 12038095\,mm^5 \cdot \frac{1}{a^4}$$

$$f_\zeta = \frac{18970\,N \cdot (4000\,mm)^3}{192 \cdot 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot \frac{10}{3} \cdot a^4} = 9033333\,mm^5 \cdot \frac{1}{a^4}$$

$$f = \sqrt{f_\eta^2 + f_\zeta^2} = \sqrt{\left(\frac{12038095\,mm^5}{a^4}\right)^2 + \left(\frac{9033333\,mm^5}{a^4}\right)^2}$$

$$f = \frac{1}{a^4} \cdot 15050476\,mm^5 = 5\,mm$$

$$a = \sqrt[4]{\frac{15050476\,mm^5}{5\,mm}} = 41{,}65\,mm$$

2) $y_A = 0{,}25 \cdot 50\,mm = 12{,}5\,mm = 1{,}25\,cm$

$z_A = 1{,}75 \cdot 50\,mm = 87{,}5\,mm = 8{,}75\,cm$

$$\eta_A = 8{,}75\,cm \cdot \sin(18{,}43^\circ) + 1{,}25\,cm \cdot \cos(18{,}43^\circ) = 3{,}95\,cm$$

$$\zeta_A = 8{,}75\,cm \cdot \cos(18{,}43^\circ) - 1{,}25\,cm \cdot \sin(18{,}43^\circ) = 7{,}91\,cm$$

$$\alpha = -\varphi^* = -18{,}43^\circ$$

$$M_{by} = M_{bY} \cdot \cos\alpha = 10\,kN\cdot m \cdot \cos(-18{,}43°) = 9{,}49\,kN\cdot m$$

$$M_{bz} = M_{bY} \cdot \sin\alpha = 10\,kN\cdot m \cdot \sin(-18{,}43°) = -3{,}16\,kN\cdot m$$

$$I_y = \frac{10}{3} \cdot (5\,cm)^4 = 2083\,cm^4$$

$$I_z = \frac{25}{30} \cdot (5\,cm)^4 = 521\,cm^4$$

$$\sigma_b^A = \frac{949\,kN\cdot cm}{2083\,cm^4} \cdot 7{,}91\,cm - \frac{(-316\,kN\cdot cm)}{521\,cm^4} \cdot 3{,}95\,cm$$

$$\sigma_b^A = 6\,\frac{kN}{cm^2} = 60\,\frac{N}{mm^2}$$

1/

$S_{y1} = 0$

$S_{y2} = 9\,cm \cdot 1\,cm \cdot 9{,}3\,cm = 83{,}7\,cm^3$

$S_{y3} = 9\,cm \cdot 3\,cm \cdot 8{,}3\,cm - \frac{\pi \cdot (2\,cm)^2}{2} \cdot \left(6{,}80\,cm + \frac{4 \cdot 2\,cm}{3\pi}\right)$

$S_{y3} = 176{,}04\,cm^3$

$S_{y4} = 9\,cm \cdot 5\,cm \cdot 7{,}3\,cm - \pi \cdot (2\,cm)^2 \cdot 6{,}80\,cm = 243{,}05\,cm^3$

$S_{y5} = 243{,}05\,cm^3 + 3\,cm \cdot 3\,cm \cdot 3{,}3\,cm + 2 \cdot \frac{3\,cm \cdot 3\,cm}{2} \cdot 3{,}8\,cm$

$S_{y5} = 306{,}95\,cm^3$

$S_{y6} = 306{,}95\,cm^3 + 3\,cm \cdot 4\,cm \cdot (-0{,}20\,cm) = 304{,}55\,cm^3$

$S_{y7} = 304{,}55\,cm^3 + 3\,cm \cdot 3\,cm \cdot (-3{,}7\,cm) + 2 \cdot \frac{3\,cm \cdot 4\,cm}{2} \cdot (-4{,}2\,cm)$

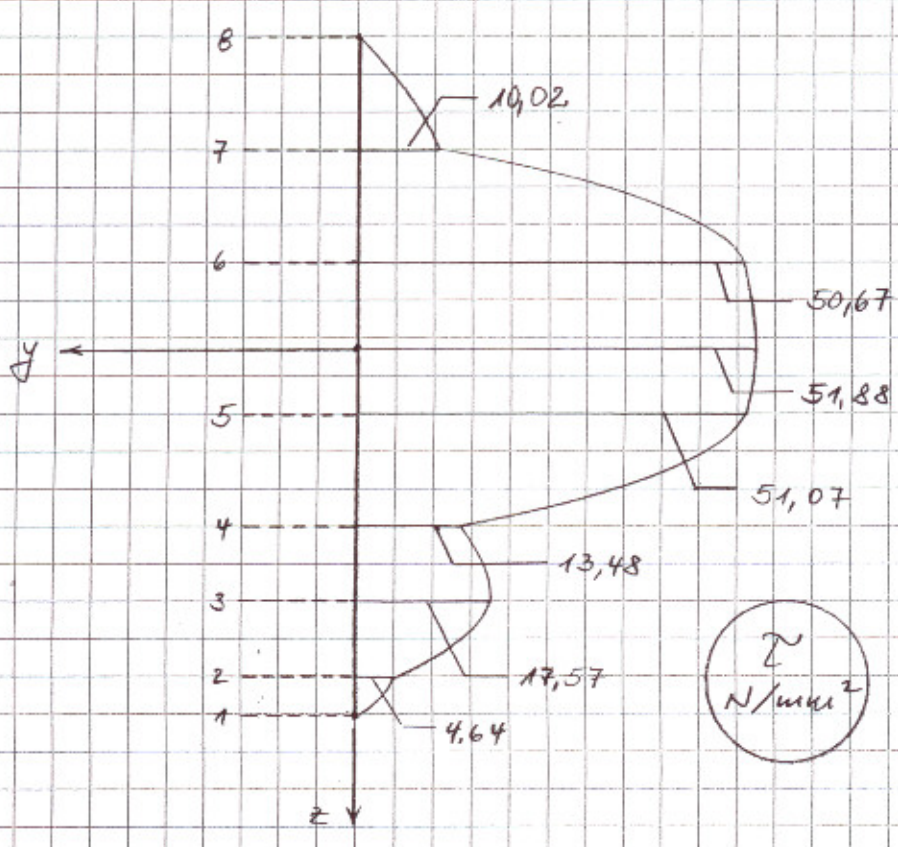
$S_{y7} = 220{,}85\,cm^3$

$S_{y8} = 220{,}85\,cm^3 + 11\,cm \cdot 3\,cm \cdot (-6{,}7\,cm) = -0{,}25\,cm^3 \approx 0$

Schnitt	S_y cm^3	b cm	τ kN/cm^2	τ N/mm^2
1	0	9	0	0
2	83,7	9	0,464	4,64
3	176,04	5	1,757	17,57
4	243,05	9	1,348	13,48
5	306,95	3	5,107	51,07
6	304,55	3	5,067	50,67
7	220,85	11	1,002	10,02
8	0	11	0	0

2) $S_{ymax} = 306{,}95\,cm^3 + 3\,cm \cdot 1{,}8\,cm \cdot 0{,}9\,cm = 311{,}81\,cm^3$

$\tau_{max} = 51{,}88\ \frac{N}{mm^2}$ in der Faser $z = 0$



$$\frac{K_1}{K_2} = \sqrt{\frac{F}{E \cdot I_1} \cdot \frac{E \cdot I_2}{F}} = \sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{d_2^4}{d_1^4}} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$K_2 = \frac{9}{4} \cdot K_1$$

$$\ell_2 = K_2 \cdot l_2 = \frac{9}{4} \cdot K_1 \cdot \frac{1}{2} \cdot l_1 = \frac{9}{8} \cdot \varkappa_1$$

Knickbedingung

$$f(\varkappa_1) = \tan \varkappa_1 + \frac{4}{9} \cdot \tan\left(\frac{9}{8} \cdot \varkappa_1\right) = 0$$

1. Iteration

$$f(1{,}42) = -10{,}058$$

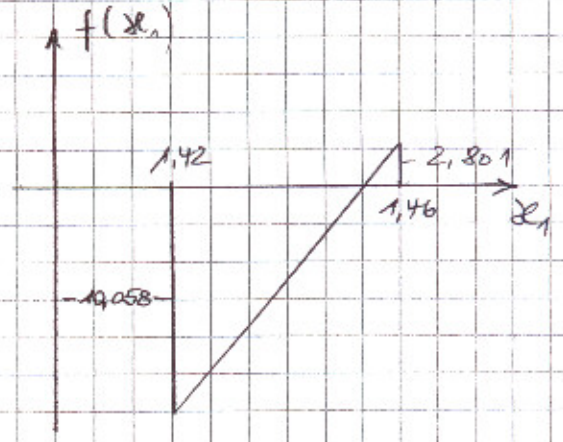
$$f(1{,}46) = 2{,}801$$

$$\frac{\Delta \varkappa_1}{10{,}058} = \frac{0{,}04}{10{,}058 + 2{,}801}$$

$$\Delta \varkappa_1 = 0{,}031$$

$$\varkappa_{1\,Neu} = 1{,}42 + 0{,}031 = 1{,}451$$

$$f(1{,}451) = 1{,}099$$



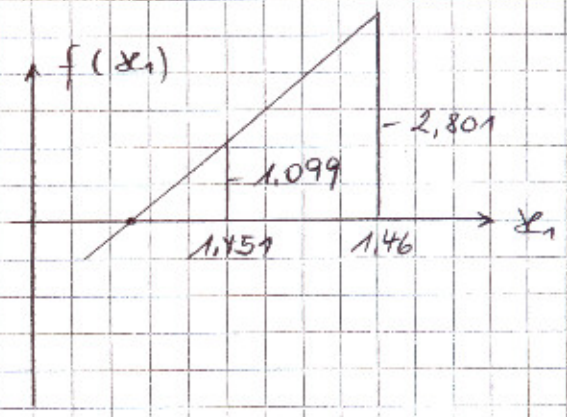
2. Iteration

$$\frac{\Delta \varkappa_1}{2{,}801} = \frac{1{,}46 - 1{,}451}{2{,}801 - 1{,}099}$$

$$\Delta \varkappa_1 = 0{,}015$$

$$\varkappa_{1\,Neu} = 1{,}46 - 0{,}015 = 1{,}445$$

$$f(1{,}445) = -0{,}19 \sim 0$$



$$\sqrt{\frac{F_K}{E \cdot I_1}} \cdot l_1 = 1{,}445$$

$$\frac{F_K}{E \cdot \frac{\pi \cdot d_1^4}{64}} \cdot l_1^2 = 1{,}445^2$$

$$d_1^4 = \frac{64 \cdot \nu \cdot F \cdot l_1^2}{E \cdot \pi \cdot 1{,}445^2}$$

$$d_1 = \sqrt[4]{\frac{64 \cdot 4 \cdot 50000\,N \cdot (2000\,mm)^2}{2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot \pi \cdot 1{,}445^2}}$$

$$d_1 = 78\,mm$$

2) $$A_2 = \frac{\pi \cdot d_2^2}{4} = \frac{\pi \cdot \left(\frac{2}{3} \cdot d_1\right)^2}{4} = \frac{\pi \cdot \frac{4}{9} \cdot (78\,mm)^2}{4}$$

$$A_2 = 2124\,mm^2$$

$$\sigma_K = \frac{4 \cdot 50000\,N}{2124\,mm^2} = 94{,}16\,\frac{N}{mm^2} < 188\,\frac{N}{mm^2} = \sigma_{dP}$$