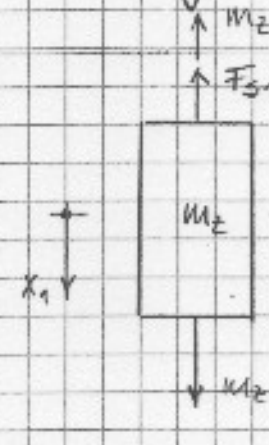
1/ a) Zylinder

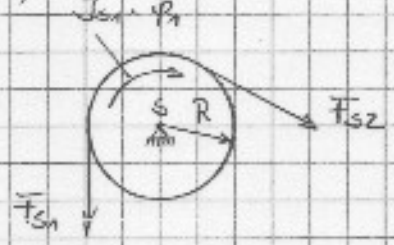


$\Sigma F_{iv} = 0$

$m_z \cdot \ddot{x}_1 + F_{s1} - m_z \cdot g = 0$

$F_{s1} = m_z \cdot (g - \ddot{x}_1)$ ①

b) Rolle 1



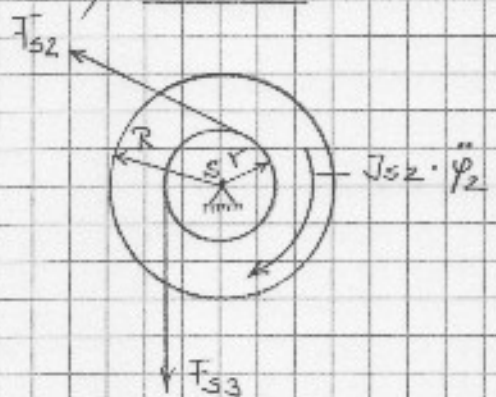
$\Sigma M_{is} = 0$

$F_{s2} \cdot R + J_{s1} \cdot \ddot{\varphi}_1 - F_{s1} \cdot R = 0$

$F_{s1} = \frac{J_{s1}}{R} \cdot \ddot{\varphi}_1 + F_{s2}$ ②

Kinematik: $\ddot{\varphi}_1 = \frac{\ddot{x}_1}{R}$ ③

c) Rolle 2



$\Sigma M_{is} = 0$

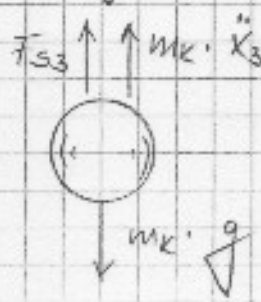
$F_{s2} \cdot r + F_{s3} \cdot r - J_{s2} \cdot \ddot{\varphi}_2 = 0$

$F_{s2} = \frac{J_{s2}}{r} \cdot \ddot{\varphi}_2 - F_{s3}$ ④

Kinematik: $\ddot{\varphi}_2 = \frac{\ddot{x}_1}{r} = \frac{\ddot{x}_3}{r}$ ⑤

$\ddot{x}_1 = \ddot{x}_3$ ⑥

d) Kugel



$\Sigma F_{iv} = 0$

$F_{s3} + m_K \cdot \ddot{x}_3 - m_K \cdot g = 0$

$F_{s3} = m_K \cdot (g - \ddot{x}_3)$ ⑦

Lösung

(1) = (2)

$$m_2 \cdot (g - \ddot{x}_1) = \frac{J_{S1}}{R} \cdot \ddot{\varphi}_1 + F_{S2} \quad (8)$$

(7) in (4)

$$F_{S2} = \frac{J_{S2}}{r} \cdot \ddot{\varphi}_2 - m_K \cdot (g - \ddot{x}_3)$$

Einsetzen in (8)

$$m_2 \cdot (g - \ddot{x}_1) = \frac{J_{S1}}{R} \cdot \ddot{\varphi}_1 + \frac{J_{S2}}{r} \cdot \ddot{\varphi}_2 - m_K \cdot (g - \ddot{x}_3)$$

Mit (3), (5) und (6)

$$m_2 \cdot g - m_2 \cdot \ddot{x}_1 = \frac{J_{S1}}{R^2} \cdot \ddot{x}_1 + \frac{J_{S2}}{r^2} \cdot \ddot{x}_1 - m_K \cdot g + m_K \cdot \ddot{x}_1$$

$$\left(m_2 + m_K + \frac{J_{S1}}{R^2} + \frac{J_{S2}}{r^2}\right) \cdot \ddot{x}_1 = m_2 \cdot g + m_K \cdot g$$

$$\ddot{x}_1 = \frac{m_2 \cdot g + m_K \cdot g}{m_2 + m_K + \frac{J_{S1}}{R^2} + \frac{J_{S2}}{r^2}}$$

$$J_{S1} = \frac{1}{2} \cdot m_1 \cdot R^2 = \frac{1}{2} \cdot 60\,kg \cdot (0{,}2\,m)^2 = 1{,}2\,kg \cdot m^2$$

$$J_{S2} = \frac{1}{2} \cdot m_2 \cdot R^2 = \frac{1}{2} \cdot 75\,kg \cdot (0{,}2\,m)^2 = 1{,}5\,kg \cdot m^2$$

$$\ddot{x}_1 = \frac{20\,kg \cdot 9{,}81\,\frac{m}{s^2} + 10\,kg \cdot 9{,}81\,\frac{m}{s^2}}{20\,kg + 10\,kg + \frac{1{,}2\,kg\,m^2}{(0{,}2\,m)^2} + \frac{1{,}5\,kg\,m^2}{(0{,}1\,m)^2}}$$

$$\ddot{x}_1 = 1{,}401\,\frac{m}{s^2}$$

$$\ddot{x}_3 = 1{,}401\,\frac{m}{s^2}$$

2) $F_{S1} = m_2 \cdot (g - \ddot{x}_1) = 20\,kg \cdot (9{,}81\,\frac{m}{s^2} - 1{,}401\,\frac{m}{s^2})$

$F_{S1} = 168{,}18\,N$

$F_{S2} = F_{S1} - \frac{J_{S1}}{R} \cdot \ddot{\varphi}_1$

$F_{S2} = 168{,}18\,N - \frac{1{,}2\,kg \cdot m^2}{0{,}2\,m} \cdot \frac{1{,}401\,\frac{m}{s^2}}{0{,}2\,m}$

$F_{S2} = 126{,}15\,N$

$F_{S3} = m_K \cdot (g - \ddot{x}_3) = 10\,kg \cdot (9{,}81\,\frac{m}{s^2} - 1{,}401\,\frac{m}{s^2})$

$F_{S3} = 84{,}09\,N$

3) $\dot{x}_1 = \sqrt{2 \cdot \ddot{x}_1 \cdot x_1} = \sqrt{2 \cdot 1{,}401\,\frac{m}{s^2} \cdot 2\,m} = 2{,}367\,\frac{m}{s}$

$E_K = \frac{1}{2} \cdot m_2 \cdot \dot{x}_1^2 + \frac{1}{2} \cdot J_{S1} \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot J_{S2} \cdot \dot{\varphi}_2^2 + \frac{1}{2} \cdot m_K \cdot \dot{x}_3^2$

$E_K = \frac{1}{2} \cdot 20\,kg \cdot (2{,}367\,\frac{m}{s})^2 + \frac{1}{2} \cdot 1{,}2\,kg \cdot m^2 \cdot \left(\frac{2{,}367\,\frac{m}{s}}{0{,}2\,m}\right)^2$

$+ \frac{1}{2} \cdot 1{,}5\,kg \cdot m^2 \cdot \left(\frac{2{,}367\,\frac{m}{s}}{0{,}1\,m}\right)^2 + \frac{1}{2} \cdot 10\,kg \cdot (2{,}367\,\frac{m}{s})^2$

$E_K = 56{,}03\,N \cdot m + 84{,}04\,N \cdot m + 420{,}20\,N \cdot m + 28{,}01\,N \cdot m$

$E_K = 588{,}28\,N \cdot m$

zu 1) $m \geq 2 \cdot f_N \cdot T = 2 \cdot \frac{2}{T} \cdot T = 4$

$m = 4$ Stützstellen erforderlich

zu 2) $a_0 = \frac{1}{m} \cdot \sum_{i=0}^{m-1} F(t_i)$

$a_k = \frac{2}{m} \cdot \sum_{i=0}^{m-1} F(t_i) \cdot \cos\left(\frac{2 \cdot k \cdot i \cdot \pi}{m}\right)$

i	t_i s	$F(t_i)$ N	$\cos\left(\frac{2 \cdot i \cdot \pi}{5}\right)$ -	$\cos\left(\frac{4 \cdot i \cdot \pi}{5}\right)$ -	$F(t_i) \cdot \cos\left(\frac{2 \cdot i \cdot \pi}{5}\right)$ N	$F(t_i) \cdot \cos\left(\frac{4 \cdot i \cdot \pi}{5}\right)$ N
0	0	100,000	1,0000	1,0000	100,000	100,000
1	0,02	109,310	0,3090	-0,8090	33,777	-88,432
2	0,04	148,459	-0,8090	0,3090	-120,103	45,874
3	0,06	-86,655	-0,8090	0,3090	70,104	-26,776
4	0,08	-271,113	0,3090	-0,8090	-83,774	219,330
		0			0	250

$a_0 = 0$; $a_1 = 0$; $a_2 = \frac{2}{5} \cdot 250\,N = 100\,N$

$b_k = \frac{2}{m} \cdot \sum_{i=0}^{m-1} F(t_i) \cdot \sin\left(\frac{2 \cdot k \cdot i \cdot \pi}{5}\right)$

i	t_i s	$F(t_i)$ N	$\sin\left(\frac{2 \cdot i \cdot \pi}{5}\right)$ -	$\sin\left(\frac{4 \cdot i \cdot \pi}{5}\right)$ -	$F(t_i) \cdot \sin\left(\frac{2 \cdot i \cdot \pi}{5}\right)$ N	$F(t_i) \cdot \sin\left(\frac{4 \cdot i \cdot \pi}{5}\right)$ N
0	0	100,000	0	0	0	0
1	0,02	109,310	0,9511	0,5878	103,965	64,252
2	0,04	148,459	0,5878	-0,9511	87,264	-141,199
3	0,06	-86,655	-0,5878	0,9511	50,936	-82,418
4	0,08	-271,113	-0,9511	-0,5878	257,856	159,360
					500	0

$b_1 = \frac{2}{5} \cdot 500\,N = 200\,N$

$b_2 = 0$

$F(t) = 200\,N \cdot \sin\left(\frac{2\pi}{T} \cdot t\right) + 100\,N \cdot \cos\left(\frac{4\pi}{T} \cdot t\right)$

$F(t) = 200\,N \cdot \sin\left(20\pi \frac{1}{s} \cdot t\right) + 100\,N \cdot \cos\left(40\pi \frac{1}{s} \cdot t\right)$

zu 3) $A_1 = \sqrt{a_1^2 + b_1^2} = \sqrt{(200\,N)^2} = 200\,N$

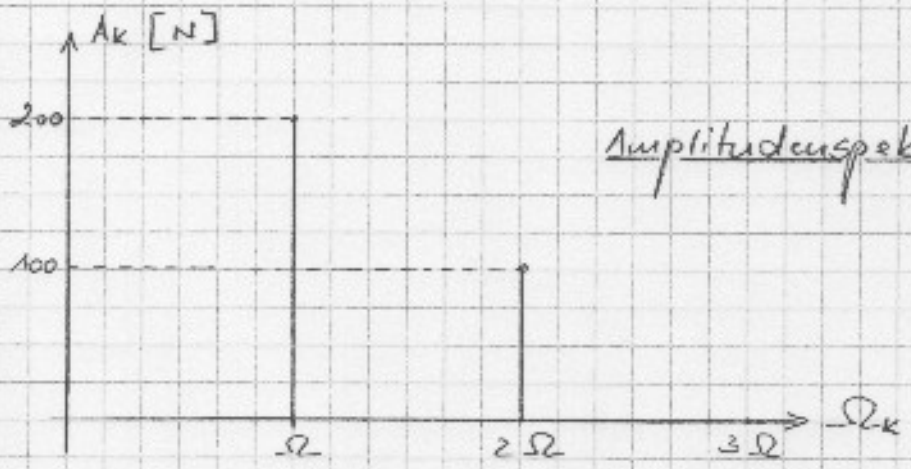
$A_2 = \sqrt{a_2^2 + b_2^2} = \sqrt{(100\,N)^2} = 100\,N$

$\tan \varphi_1 = \frac{a_1}{b_1} = \frac{0}{200N} = 0 \Rightarrow \varphi_1 = 0$

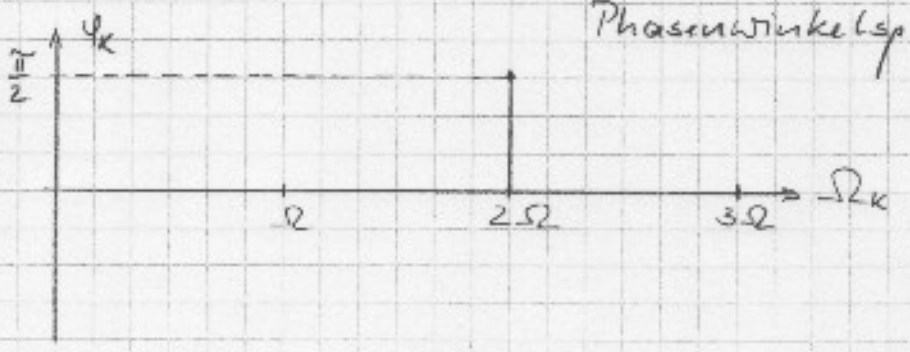
$\tan \varphi_2 = \frac{a_2}{b_2} = \frac{100N}{0} \to \infty \Rightarrow \varphi_2 = \frac{\pi}{2}$

$F(t) = 200\,N \cdot \sin\left(20\pi \frac{1}{s} \cdot t\right) + 100N \cdot \sin\left(40\pi \frac{1}{s} \cdot t + \frac{\pi}{2}\right)$

zu 4)



Amplitudenspektrum



Phasenwinkelspektrum

zu 1)

a) $\Lambda = \frac{1}{4} \cdot \ln\left(\frac{5\,mm}{2\,mm}\right) = 0,229$ (1)

$$D = \frac{\Lambda}{\sqrt{4 \cdot \pi^2 + \Lambda^2}} = \frac{0,229}{\sqrt{4\pi^2 + 0,229^2}} = 0,036$$ (1)

b) $\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{0,12s} = 52,36\ \frac{1}{s}$ (1)

c) $\omega_0 = \frac{\omega_d}{\sqrt{1-D^2}} = \frac{52,36\ \frac{1}{s}}{\sqrt{1-0,036^2}} = 52,39\ \frac{1}{s}$ (1)

d) $C_1 = W_0 = 5\,mm$ (1)

$$\delta = D \cdot \omega_0 = 0,036 \cdot 52,39\ \frac{1}{s} = 1,886\ \frac{1}{s}$$

$$C_2 = \frac{\delta \cdot W_0}{\omega_d} = \frac{1,886\ \frac{1}{s} \cdot 5\,mm}{52,36\ \frac{1}{s}} = 0,180\,mm$$ (1)

$$W(t) = e^{-1,886\ \frac{1}{s} \cdot t} \cdot \left[5\,mm \cdot \cos\left(52,36\ \frac{1}{s} \cdot t\right) + 0,180\,mm \cdot \sin\left(52,36\ \frac{1}{s} \cdot t\right)\right]$$ (1)

e) $$c = \frac{12 \cdot 2,1 \cdot 10^5\ \frac{N}{mm^2} \cdot 328 \cdot 10^4\,mm^4}{(1000\,mm)^3 \cdot (4 \cdot 1000\,mm + 3 \cdot 3000\,mm)}$$

$$c = 635,815\ \frac{N}{mm} = 635815\ \frac{N}{m}$$ (1)

f) $\omega_0 = \sqrt{\frac{c}{m}}$

$$m = \frac{c}{\omega_0^2} = \frac{635815\ \frac{N}{m}}{(52,39\ \frac{1}{s})^2} = 231,65\,kg$$ (1)

zu 2)

a) $\Omega = \frac{2\pi \cdot 600\,\text{min}^{-1}}{60\,\frac{s}{\text{min}}} = 62,83\,\frac{1}{s}$ (1)

$\eta = \frac{62,83\,\frac{1}{s}}{52,39\,\frac{1}{s}} = 1,20$ (1)

$$W_{max} = \frac{1,20^2}{\sqrt{(1-1,2^2)^2 + 4 \cdot 0,036^2 \cdot 1,2^2}} \cdot \frac{0,4\,kg \cdot m}{231,65\,kg}$$

$W_{max} = 5,55 \cdot 10^{-3}\,m = 5,55\,mm$ (1)

$$\varphi = -\arctan\left(\frac{2 \cdot 0,036 \cdot 1,2}{1 - 1,2^2}\right)$$

$\varphi = -168,89° = -2,948$ (1)

$W(t) = 5,55\,mm \cdot \sin\left(62,83\,\frac{1}{s} \cdot t - 2,948\right)$ (1)

b) $F = c \cdot W_{max} = 635,815\,\frac{N}{mm} \cdot 5,55\,mm = 3529\,N$

$M = -F \cdot b = -3529\,N \cdot 1000\,mm = -3529 \cdot 10^3\,N \cdot mm$ (1)

$\sigma = \frac{3529 \cdot 10^3\,N \cdot mm}{54,7 \cdot 10^3\,mm^3} = 64,51\,\frac{N}{mm^2}$ (1)