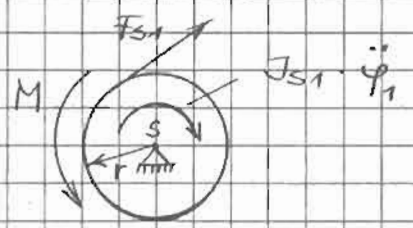
zu 1)

Rolle 1

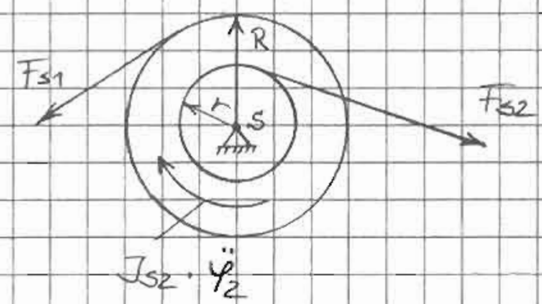


$$\sum M_{iS} = 0$$

$$F_{S1}\cdot r + J_{S1}\cdot\ddot{\varphi}_1 - M = 0$$

$$F_{S1} = \frac{M}{r} - \frac{J_{S1}}{r}\cdot\ddot{\varphi}_1 \qquad (1)$$

Rolle 2

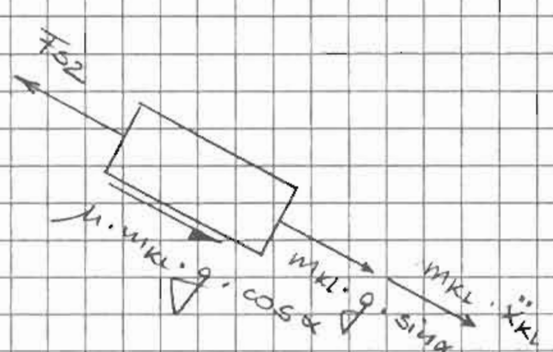


$$\sum M_{iS} = 0$$

$$F_{S2}\cdot r + J_{S2}\cdot\ddot{\varphi}_2 - F_{S1}\cdot R = 0$$

$$F_{S1} = F_{S2}\cdot\frac{r}{R} + \frac{J_{S2}}{R}\cdot\ddot{\varphi}_2 \qquad (2)$$

Klotz



$$\sum F_{it} = 0$$

$$F_{S2} - m_{KL}\cdot g\cdot\sin\alpha - m_{KL}\cdot\ddot{x}_{KL}$$
$$- \mu\cdot m_{KL}\cdot g\cdot\cos\alpha = 0$$

$$F_{S2} = m_{KL}\cdot\ddot{x}_{KL} + m_{KL}\cdot g\cdot(\sin\alpha + \mu\cdot\cos\alpha) \qquad (3)$$

Kinematik

$$x_{KL} = r\cdot\varphi_2\ ; \quad \varphi_2 = \frac{x_{KL}}{r} \qquad (4)$$

$$\varphi_1\cdot r = \varphi_2\cdot R$$

$$\varphi_1 = \varphi_2\cdot\frac{R}{r} = x_{KL}\cdot\frac{R}{r^2}\ ; \quad x_{KL} = \varphi_1\cdot\frac{r^2}{R} \qquad (5)$$

① = ②

$$\frac{M}{r} - \frac{J_{S1}}{r} \cdot \ddot{\varphi}_1 = F_{S2} \cdot \frac{r}{R} + \frac{J_{S2}}{R} \cdot \ddot{\varphi}_2$$

③ einsetzen

$$\frac{M}{r} - \frac{J_{S1}}{r} \cdot \ddot{\varphi}_1 = m_{KL} \cdot \frac{r}{R} \cdot \ddot{x}_{KL} + m_{KL} \cdot g \cdot \frac{r}{R} \cdot (\sin\alpha + \mu \cdot \cos\alpha)$$
$$+ \frac{J_{S2}}{R} \cdot \ddot{\varphi}_2$$

mit ④ und ⑤

$$\frac{M}{r} - \frac{\frac{1}{2} \cdot m_1 \cdot r^2}{r} \cdot \ddot{x}_{KL} \cdot \frac{R}{r^2} = m_{KL} \cdot \frac{r}{R} \cdot \ddot{x}_{KL} + \frac{\frac{1}{2} \cdot m_2 \cdot R^2}{R} \cdot \frac{\ddot{x}_{KL}}{r}$$
$$+ m_{KL} \cdot g \cdot \frac{r}{R} \cdot (\sin\alpha + \mu \cdot \cos\alpha)$$

$$\left(m_{KL} \cdot \frac{r}{R} + \frac{1}{2} \cdot m_1 \cdot \frac{R}{r} + \frac{1}{2} \cdot m_2 \cdot \frac{R}{r}\right) \cdot \ddot{x}_{KL} = \frac{M}{r} - m_{KL} \cdot g \cdot \frac{r}{R} \cdot$$
$$(\sin\alpha + \mu \cdot \cos\alpha)$$

$$\left(\frac{1}{2} \cdot m_{KL} + m_1 + m_2\right) \cdot \ddot{x}_{KL} = \frac{M}{r} - \frac{1}{2} \cdot m_{KL} \cdot g \cdot (\sin\alpha + \mu \cdot \cos\alpha)$$

$$\ddot{x}_{KL} = \frac{\frac{M}{r} - \frac{1}{2} \cdot m_{KL} \cdot g \cdot (\sin\alpha + \mu \cdot \cos\alpha)}{\frac{1}{2} \cdot m_{KL} + m_1 + m_2}$$

$$\ddot{x}_{KL} = \frac{\frac{100\,N \cdot m}{0{,}15\,m} - \frac{1}{2} \cdot 100\,kg \cdot 9{,}81\,\frac{m}{s^2} \cdot (\sin 30° + 0{,}2 \cdot \cos 30°)}{50\,kg + 30\,kg + 60\,kg}$$

$$\ddot{x}_{KL} = 2{,}403\ \frac{m}{s^2}$$

2) $x_{KL} = \varphi_1 \cdot \frac{r^2}{R} = 5 \cdot 2\pi \cdot \frac{(0{,}15\,m)^2}{0{,}3\,m} = 2{,}36\,m$

$$\dot{x}_{KL} = \sqrt{2 \cdot \ddot{x}_{KL} \cdot x_{KL}} = \sqrt{2 \cdot 2{,}403\,\frac{m}{s^2} \cdot 2{,}36\,m}$$

$$\dot{x}_{KL} = 3{,}37\ \frac{m}{s}$$

$$\varphi_2 = \varphi_1 \cdot \frac{r}{R} = 5 \cdot 2\pi \cdot \frac{1}{2} = 5\pi$$

$$n_2 = \frac{1}{2} \cdot n_1 = 2{,}5 \text{ Umdrehungen}$$

3)

$$E_K = \frac{1}{2} \cdot m_{KL} \cdot \dot{x}_{KL}^2 + \frac{1}{2} \cdot J_{S1} \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot J_{S2} \cdot \dot{\varphi}_2^2$$

$$E_K = \frac{1}{2} \cdot m_{KL} \cdot \dot{x}_{KL}^2 + \frac{1}{2} \cdot \frac{1}{2} \cdot m_1 \cdot \cancel{r^2} \cdot \frac{\dot{x}_{KL}^2}{\cancel{r^2}} \cdot R^2$$

$$+ \frac{1}{2} \cdot \frac{1}{2} \cdot m_2 \cdot R^2 \cdot \frac{\dot{x}_{KL}^2}{r^2}$$

$$E_K = \left(\frac{1}{2} \cdot m_{KL} + m_1 + m_2\right) \cdot \dot{x}_{KL}^2$$

$$E_K = \left(\frac{1}{2} \cdot m_{KL} + m_1 + m_2\right) \cdot 2 \cdot \ddot{x}_{KL} \cdot x_{KL}$$

$$E_K = (50\,kg + 30\,kg + 60\,kg) \cdot 2 \cdot 2{,}403 \frac{m}{s^2} \cdot x_{KL}$$

$$E_K = 672{,}84\,N \cdot x_{KL}$$

zu 1) $\underset{\smile}{y}(t) = \underset{\smile}{y}(-t) \rightarrow \underset{\smile}{y}(t)$ gerade Funktion $\rightarrow b_k = 0$ (1)

a) Fourierkoeffizient a_0

$$a_0 = \frac{2}{T} \cdot \int_0^{\frac{T}{2}} \underset{\smile}{y}(t) \cdot dt = \frac{2}{T} \cdot \int_0^{\frac{T}{4}} h \cdot dt + \frac{2}{T} \cdot \int_{\frac{T}{4}}^{\frac{T}{2}} \left(\frac{12 \cdot h}{T} \cdot t - 2 \cdot h \right) \cdot dt$$

$$a_0 = \frac{2 \cdot h}{T} \cdot t \Big|_0^{\frac{T}{4}} + \frac{24 \cdot h}{T^2} \cdot \frac{t^2}{2} \Big|_{\frac{T}{4}}^{\frac{T}{2}} - \frac{4 \cdot h}{T} \cdot t \Big|_{\frac{T}{4}}^{\frac{T}{2}}$$

$$a_0 = \frac{2 \cdot h}{T} \cdot \frac{T}{4} + \frac{24 \cdot h}{T^2} \cdot \left(\frac{T^2}{8} - \frac{T^2}{32} \right) - \frac{4 \cdot h}{T} \cdot \left(\frac{T}{2} - \frac{T}{4} \right)$$

$$a_0 = \frac{h}{2} + \frac{9}{4} \cdot h - h = \frac{9}{4} \cdot h - \frac{h}{2} = \frac{7}{4} \cdot h \qquad (2)$$

b) Fourierkoeffizienten a_k

$$a_k = \frac{4}{T} \cdot \int_0^{\frac{T}{4}} \underset{\smile}{y}(t) \cdot \cos(k \cdot \omega \cdot t) \cdot dt$$

$$a_k = \frac{4}{T} \cdot \int_0^{\frac{T}{4}} h \cdot \cos(k \cdot \omega \cdot t) \cdot dt + \frac{4}{T} \cdot \int_{\frac{T}{4}}^{\frac{T}{2}} \left(\frac{12 \cdot h}{T} \cdot t - 2 \cdot h \right) \cdot \cos(k \cdot \omega \cdot t) \cdot dt$$

$$a_k = \frac{4 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \sin(k \cdot \omega \cdot t) \Big|_0^{\frac{T}{4}}$$

$$+ \frac{48 \cdot h}{T^2} \cdot \left[\frac{\cos(k \cdot \omega \cdot t)}{k^2 \cdot \omega^2} + \frac{t \cdot \sin(k \cdot \omega \cdot t)}{k \cdot \omega} \right]_{\frac{T}{4}}^{\frac{T}{2}}$$

$$- \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \sin(k \cdot \omega \cdot t) \Big|_{\frac{T}{4}}^{\frac{T}{2}}$$

$$a_k = \frac{4\cdot h}{T}\cdot\frac{1}{k\cdot\omega}\cdot\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)$$

$$+\frac{48\cdot h}{T^2}\cdot\left[\frac{\cos\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{k^2\cdot\omega^2}+\frac{\frac{T}{2}\cdot\overset{0}{\cancel{\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{2}\right)}}}{k\cdot\omega}\right.$$

$$\left.-\frac{\cos\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{k^2\cdot\omega^2}-\frac{T}{4}\cdot\frac{\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{k\cdot\omega}\right]$$

$$-\frac{8\cdot h}{T}\cdot\frac{1}{k\cdot\omega}\cdot\left[\overset{0}{\cancel{\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{2}\right)}}-\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)\right]$$

$$a_k = \cancel{\frac{4\cdot h}{T}\cdot\frac{1}{k\cdot\omega}\cdot\sin\left(k\cdot\frac{\pi}{2}\right)}$$

$$+\frac{48\cdot h}{T^2}\cdot\frac{\cos(k\cdot\pi)}{k^2\cdot\omega^2}-\frac{48\cdot h}{T^2}\cdot\frac{\cos\left(k\cdot\frac{\pi}{2}\right)}{k^2\cdot\omega^2}$$

$$\cancel{-\frac{12\cdot h}{T}\cdot\frac{\sin\left(k\cdot\frac{\pi}{2}\right)}{k\cdot\omega}}+\cancel{\frac{8\cdot h}{T}\cdot\frac{\sin\left(k\cdot\frac{\pi}{2}\right)}{k\cdot\omega}}$$

$$a_k = \frac{48\cdot h}{k^2\cdot T^2}\cdot\frac{1}{\frac{4\pi^2}{T^2}}\cdot\left[\cos(k\cdot\pi)-\cos\left(\frac{k\cdot\pi}{2}\right)\right]$$

$$a_k = \frac{12\cdot h}{k^2\cdot\pi^2}\cdot\left[(-1)^k-\cos\left(\frac{k\cdot\pi}{2}\right)\right] \qquad 6$$

$$a_1 = \frac{12\cdot h}{1^2\cdot\pi^2}\cdot\left[-1-0\right] = -1{,}216\cdot h\,;\quad a_2 = \frac{12\cdot h}{2^2\cdot\pi^2}\cdot\left[1+1\right] = 0{,}608\cdot h$$

$$a_3 = \frac{12\cdot h}{3^2\cdot\pi^2}\cdot\left[-1-0\right] = -0{,}135\cdot h$$

2

$$a_4 = \frac{12\cdot h}{4^2\cdot\pi^2}\cdot\left[1-1\right] = 0$$

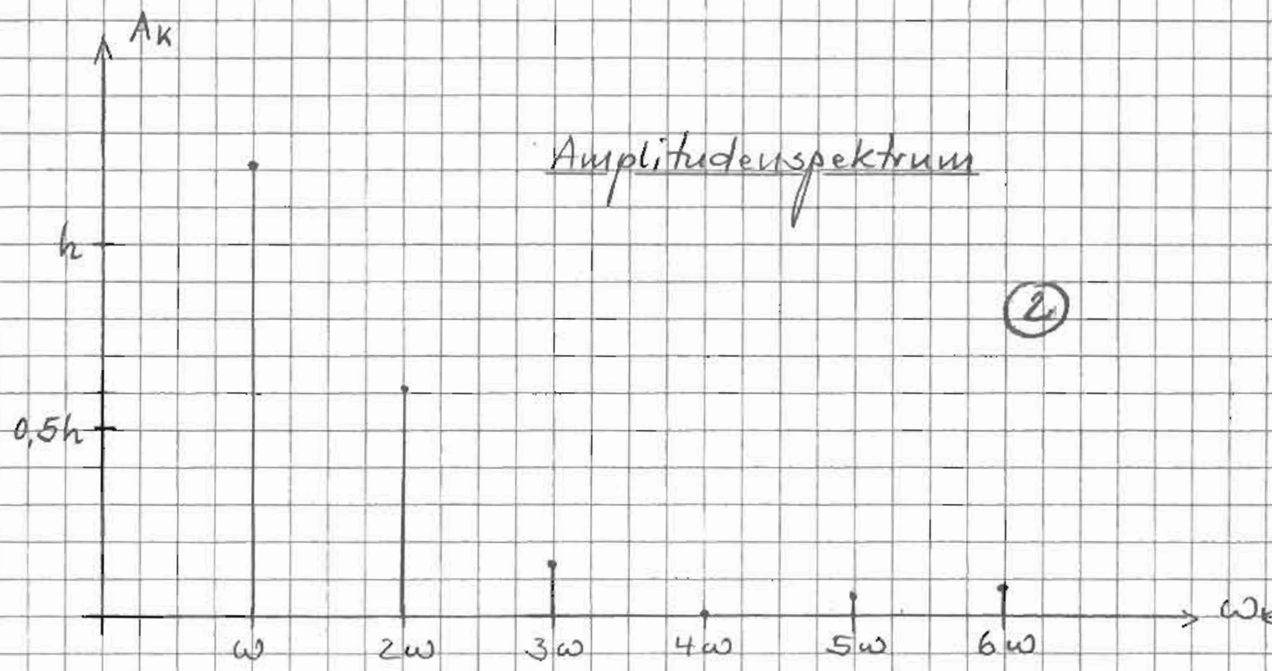
$$a_5 = \frac{12\cdot h}{5^2\cdot\pi^2}\cdot\left[-1+0\right] = -0{,}049\cdot h\,;\quad a_6 = \frac{12\cdot h}{6^2\cdot\pi^2}\cdot\left[1+1\right] = 0{,}068\cdot h$$

Fourierreihe

$$y(t) = \frac{7}{4} \cdot h - \frac{12 \cdot h}{\pi^2} \cdot \left[\cos(\omega \cdot t) - \frac{2}{2^2} \cos(2 \cdot \omega \cdot t) + \frac{1}{3^2} \cdot \cos(3 \cdot \omega \cdot t) + \frac{1}{5^2} \cdot \cos(5 \cdot \omega \cdot t) + \dots \right.$$

1

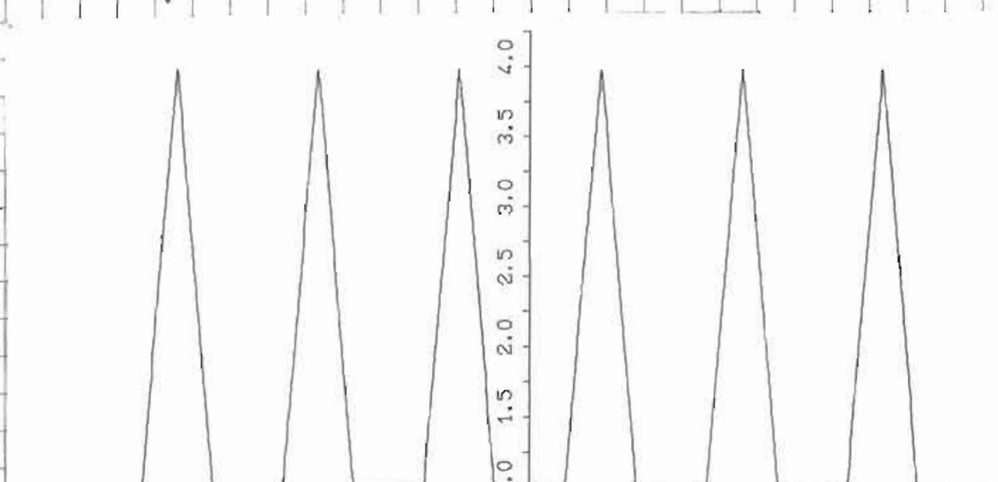
zu 2) $A_k = |a_k|$



Amplitudenspektrum

2

Auswertung



$h = 1{,}0$; 50 Harmonische, $T = 2s$

A3, 1/2

1) $\omega_0 = \sqrt{\dfrac{400\,000\ N/m}{400\ kg}} = 31{,}62\ \frac{1}{s}$

2) a) $\omega_d = \dfrac{2\pi}{T_d} = \dfrac{2\pi}{0{,}1s} = 10\pi = 31{,}42\ \frac{1}{s}$

$\omega_d = \omega_0 \cdot \sqrt{1-D^2}$

$D = \sqrt{1-\left(\dfrac{\omega_d}{\omega_0}\right)^2} = \sqrt{1-\left(\dfrac{31{,}42}{31{,}62}\right)^2} = 0{,}112$

$d = 2 \cdot D \cdot m \cdot \omega_0 = 2 \cdot 0{,}112 \cdot 400\ Kg \cdot 31{,}62\ \frac{1}{s} = 2833{,}15\ \frac{Kg}{s}$

$\delta = D \cdot \omega_0 = 0{,}112 \cdot 31{,}62\ \frac{1}{s} = 3{,}541\ \frac{1}{s}$

b) $x(t) = e^{-\delta \cdot t} \cdot \left[C_1 \cdot \cos(\omega_d \cdot t) + C_2 \cdot \sin(\omega_d \cdot t)\right]$

$C_1 = x_0 = 1\,cm = 10\,mm$

$C_2 = \dfrac{\dot{x}_0 + \delta \cdot x_0}{\omega_d} = \dfrac{3{,}541\ \frac{1}{s} \cdot 1\,cm}{31{,}42} = 0{,}113\ cm$

$= 1{,}13\ mm$

$x(t) = e^{-3{,}541 \frac{1}{s} \cdot t} \cdot \Big[10\,mm \cdot \cos\left(31{,}42\ \tfrac{1}{s} \cdot t\right)$

$+ 0{,}113\,mm \cdot \sin\left(31{,}42\ \tfrac{1}{s} \cdot t\right)\Big]$

3) $x(t) = \dfrac{F_0}{c} \cdot \left\{1 - e^{-\delta \cdot t} \cdot \left[\cos(\omega_d \cdot t) + \dfrac{D}{\sqrt{1-D^2}} \cdot \sin(\omega_d \cdot t)\right]\right\}$

$\dfrac{D}{\sqrt{1-D^2}} = \dfrac{0{,}112}{\sqrt{1-0{,}112^2}} = 0{,}113$

$\dfrac{F_0}{c} = \dfrac{2000\ N}{400\,000\ N/m} = 0{,}005\ m = 5\ mm$

A3, 2/2

$$x(t) = 5mm \cdot \left\{ 1 - e^{-3,541 \cdot t} \cdot \left[\cos\left(31,42 \tfrac{1}{s} \cdot t\right) + 0,113 \cdot \sin\left(31,42 \tfrac{1}{s} \cdot t\right) \right] \right\}$$

$$\tan(\omega_d \cdot t) = 0$$

$$t = \frac{\pi}{\omega_d} = \frac{\pi}{10\pi \frac{1}{s}} = 0,1 s$$

$$x(t) = 5mm \cdot \left\{ 1 - e^{-3,541 \frac{1}{s} \cdot 0,1s} \cdot \left[\cancelto{-1}{\cos(\pi)} + 0,113 \cdot \cancelto{0}{\sin(\pi)} \right] \right\}$$

$$x(t) = 5mm \cdot \left(1 + e^{-3,541 \frac{1}{s} \cdot 0,1s} \right) = 8,51 mm$$

4) $\underline{F_{ges}} = c \cdot x(t_s) + d \cdot \dot{x}(t_s)$

$$t = t_s = 0,5 s = 0,1s + 2 \cdot T_d = 2,5 \cdot T_d$$

$\Rightarrow$ bei $t = t_s$ liegt lokales Maximum vor!

$$\omega_d \cdot t = \frac{2\pi}{T_d} \cdot 2,5 \cdot T_d = 5\pi$$

$$x(t_s) = 5mm \cdot \left\{ 1 - e^{-3,541 \frac{1}{s} \cdot 0,5s} \cdot \left[\cancelto{-1}{\cos(5\pi)} + 0,113 \cdot \cancelto{0}{\sin(5\pi)} \right] \right\}$$

$$x(t_s) = 5mm \cdot \left(1 + e^{-3,541 \frac{1}{s} \cdot 0,5s} \right) = 5,85 mm \text{ (lok. Maximum)}$$

$$\dot{x}(t_s) = 0$$

$$\underline{F_{ges}} = \underline{c_{ges}} \cdot x(t_s) = 400000 \frac{N}{m} \cdot 5,85 \cdot 10^{-3} m = 2340 N$$