1) $y(t) = -y(-t)$ ungerade Funktion $\Rightarrow a_0 = a_k = 0$

$$b_k = \frac{4}{T} \cdot \int\limits_{\frac{T}{4}}^{\frac{T}{2}} \left(-\frac{4 \cdot h}{T} \cdot t + 2 \cdot h \right) \cdot \sin(k \cdot \omega \cdot t) \cdot dt$$

$$b_k = -\frac{16 \cdot h}{T^2} \cdot \int\limits_{\frac{T}{4}}^{\frac{T}{2}} t \cdot \sin(k \cdot \omega \cdot t) \cdot dt + \frac{8 \cdot h}{T} \cdot \int\limits_{\frac{T}{4}}^{\frac{T}{2}} \sin(k \cdot \omega \cdot t) \cdot dt$$

$$b_k = -\frac{16 \cdot h}{T^2} \cdot \left[\frac{\sin(k \cdot \omega \cdot t)}{k^2 \cdot \omega^2} - \frac{t \cdot \cos(k \cdot \omega \cdot t)}{k \cdot \omega} \right]_{\frac{T}{4}}^{\frac{T}{2}}$$

$$- \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \cos(k \cdot \omega \cdot t) \Big|_{\frac{T}{4}}^{\frac{T}{2}}$$

$$b_k = \frac{-16 \cdot h}{T^2} \cdot \left[\frac{\sin\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{2}\right)}{k^2 \cdot \omega^2} - \frac{\frac{T}{2} \cdot \cos\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{2}\right)}{k \cdot \omega} \right.$$

$$\left. - \frac{\sin\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{4}\right)}{k^2 \cdot \omega^2} + \frac{\frac{T}{4} \cdot \cos\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{4}\right)}{k \cdot \omega} \right]$$

$$- \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \left[\cos\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{2}\right) - \cos\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{4}\right) \right]$$

$$b_k = -\frac{16 \cdot h}{T^2} \cdot \left[\frac{\cancelto{0}{\sin(k \cdot \pi)}}{k^2 \cdot \omega^2} - \frac{\frac{T}{2} \cdot \cos(k \cdot \pi)}{k \cdot \omega} - \frac{\sin\left(\frac{k \cdot \pi}{2}\right)}{k^2 \cdot \omega^2} \right.$$

$$\left. + \frac{\frac{T}{4} \cdot \cos\left(\frac{k \cdot \pi}{2}\right)}{k \cdot \omega} \right]$$

$$- \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \left[\cos(k \cdot \pi) - \cos\left(\frac{k \cdot \pi}{2}\right) \right]$$

$$b_k = \cancel{\frac{8\cdot h}{T\cdot k\cdot \omega}\cdot \cos(k\cdot\pi)} + \frac{16\cdot h}{T^2}\cdot\frac{1}{k^2\cdot\omega^2}\cdot\sin\left(k\cdot\frac{\pi}{2}\right)$$

$$-\frac{4\cdot h}{T\cdot k\cdot\omega}\cdot\cos\left(\frac{k\cdot\pi}{2}\right) - \cancel{\frac{8\cdot h}{T\cdot k\cdot\omega}\cdot\cos(k\cdot\pi)}$$

$$+\frac{8\cdot h}{T\cdot k\cdot\omega}\cdot\cos\left(\frac{k\cdot\pi}{2}\right)$$

$$b_k = \frac{16\cdot h}{T^2}\cdot\frac{1}{k^2\omega^2}\cdot\sin\left(k\cdot\frac{\pi}{2}\right) + \frac{4\cdot h}{T\cdot k\cdot\omega}\cdot\cos\left(k\cdot\frac{\pi}{2}\right)$$

$$b_k = \frac{16\cdot h}{T^2}\cdot\frac{T^2}{k^2\cdot 4\pi^2}\cdot\sin\left(k\cdot\frac{\pi}{2}\right) + \frac{4\cdot h\cdot T}{T\cdot k\cdot 2\pi}\cdot\cos\left(k\cdot\frac{\pi}{2}\right)$$

$$b_k = \frac{4\cdot h}{k^2\cdot\pi^2}\cdot\sin\left(k\cdot\frac{\pi}{2}\right) + \frac{2\cdot h}{k\cdot\pi}\cdot\cos\left(k\cdot\frac{\pi}{2}\right)$$

$$b_1 = \frac{4\cdot h}{\pi^2} = 0{,}405\cdot h \;;\quad b_2 = -\frac{2\cdot h}{2\cdot\pi} = -0{,}318\cdot h$$

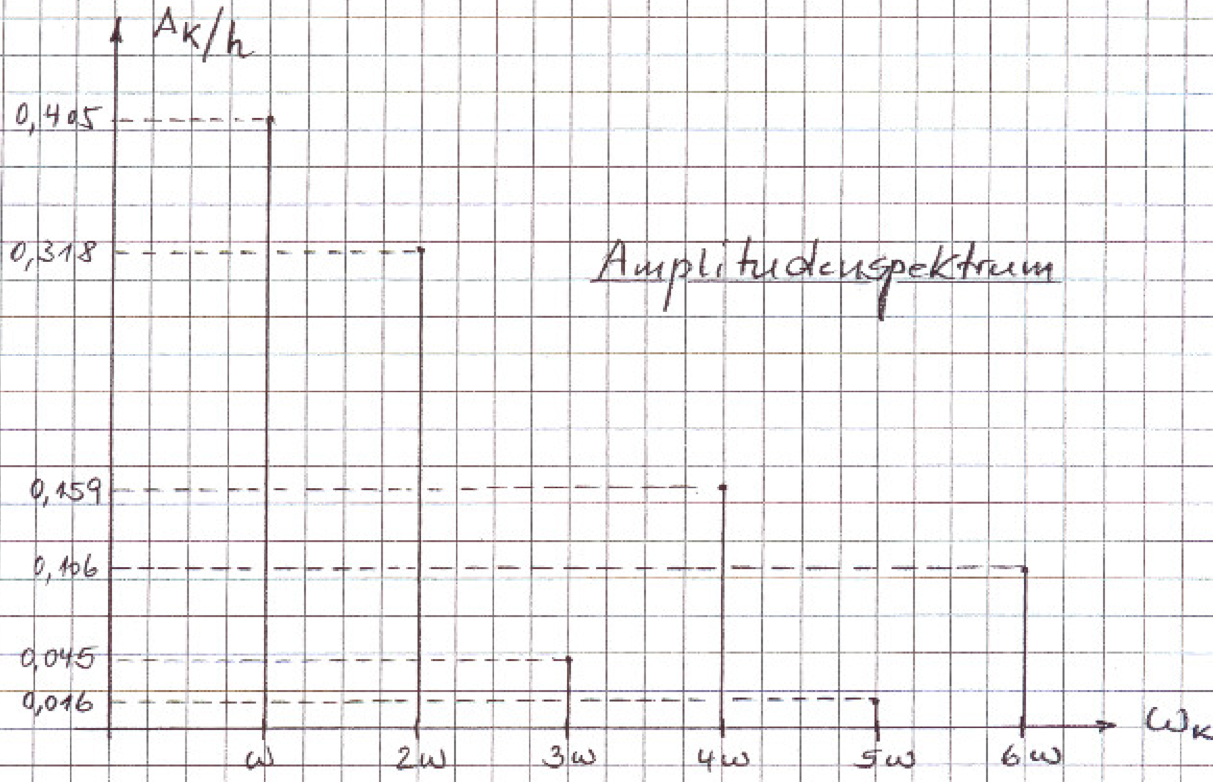
$$b_3 = -\frac{4\cdot h}{9\cdot\pi^2} = -0{,}045\cdot h \;;\quad b_4 = \frac{2\cdot h}{4\cdot\pi} = 0{,}159\cdot h$$

$$b_5 = \frac{4\cdot h}{25\cdot\pi^2} = 0{,}016\cdot h \;;\quad b_6 = -\frac{2\cdot h}{6\cdot\pi} = -0{,}106\cdot h$$

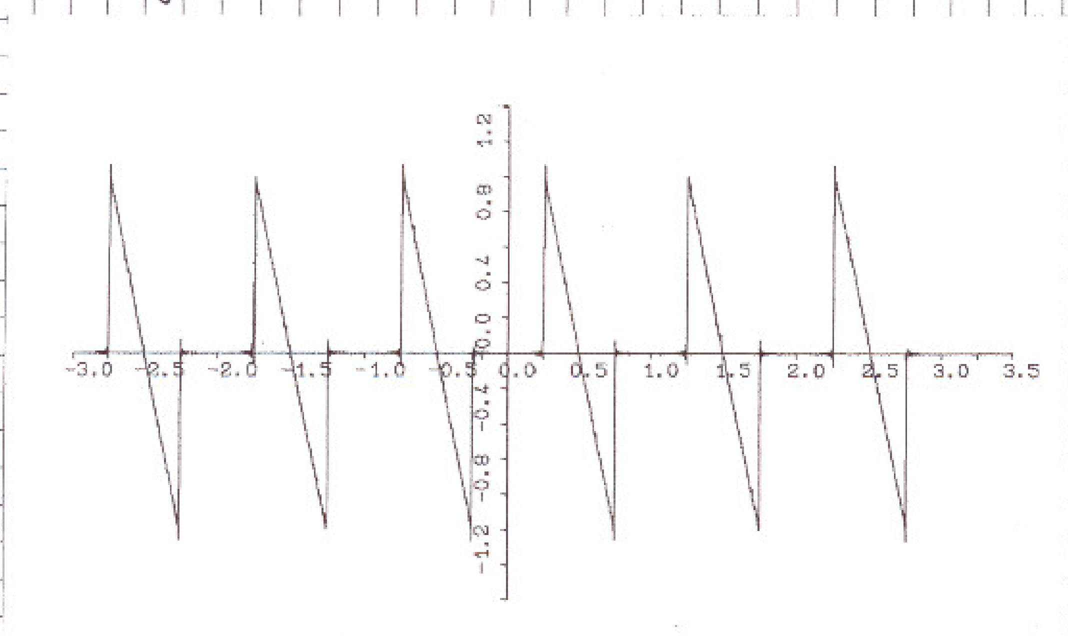
2)

$$y(t) = 0{,}405\cdot h\cdot\sin(\omega\cdot t) - 0{,}318\cdot h\cdot\sin(2\cdot\omega\cdot t)$$
$$-0{,}045\cdot h\cdot\sin(3\cdot\omega\cdot t) + 0{,}159\cdot h\cdot\sin(4\cdot\omega\cdot t)$$
$$+0{,}016\cdot h\cdot\sin(5\cdot\omega\cdot t) - 0{,}106\cdot h\cdot\sin(6\cdot\omega\cdot t) + \ldots$$

3) $A_k = \sqrt{\cancel{a_k^2}^{\,0} + b_k^2} = \sqrt{b_k^2} = |b_k|$



Auswertung: $h = 1{,}0$; $T = 1{,}0$; $K = 200$ Harmonische



zu 1)

$$c = \frac{F_{st}}{w_{st}} = \frac{m \cdot g}{w_{st}} = \frac{100\,kg \cdot 9{,}81\,\frac{m}{s^2}}{5\,mm}$$

$$c = 196{,}2\,\frac{N}{mm} = 196200\,\frac{N}{m}$$

$$\omega_0 = \sqrt{\frac{196200\,N/m}{100\,kg}} = 44{,}29\,\frac{1}{s}$$

zu 2)

$$\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{0{,}1421\,s} = 44{,}22\,\frac{1}{s}$$

$$\omega_d = \omega_0 \cdot \sqrt{1 - D^2}$$

$$D = \sqrt{1 - \left(\frac{\omega_d}{\omega_0}\right)^2} = \sqrt{1 - \left(\frac{44{,}22\,\frac{1}{s}}{44{,}29\,\frac{1}{s}}\right)^2} = 0{,}0562$$

$$\delta = D \cdot \omega_0 = 0{,}0562 \cdot 44{,}29\,\frac{1}{s} = 2{,}489\,\frac{1}{s}$$

$$\Lambda = \delta \cdot T_d = 2{,}489\,\frac{1}{s} \cdot 0{,}1421\,s = 0{,}354$$

zu 3)

$$\hat{w} = \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 4 \cdot D^2 \cdot \eta^2}} \cdot \frac{m_u \cdot r_u}{m}$$

$$0{,}006\,m = \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 4 \cdot 0{,}0562^2 \cdot \eta^2}} \cdot \frac{0{,}3\,kg \cdot m}{100\,kg}$$

$$\frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 0{,}0126 \cdot \eta^2}} = 2$$

$$\eta^4 = 4 \cdot \left[(1-\eta^2)^2 + 0{,}0126 \cdot \eta^2\right]$$

$$\eta^4 = 4 \cdot (1 - 2 \cdot \eta^2 + \eta^4 + 0{,}0126 \cdot \eta^2)$$

$$\eta^4 = 4 - 7{,}9496 \cdot \eta^2 + 4 \cdot \eta^4$$

$$3\eta^4 - 7{,}9496 \cdot \eta^2 + 4 = 0$$

$$\eta^4 - 2{,}65 \cdot \eta^2 + 1{,}\bar{3} = 0$$

$$\xi^2 - 2{,}65 \cdot \xi + 1{,}\bar{3} = 0$$

$$\xi_{1/2} = 1{,}325 \pm \sqrt{1{,}325^2 - 1{,}\bar{3}}$$

$$\xi_1 = 1{,}325 + 0{,}650 = 1{,}975$$

$$\xi_2 = 1{,}325 - 0{,}650 = 0{,}675$$

$$\eta_1 = \sqrt{1{,}975} = 1{,}405$$

$$\eta_2 = \sqrt{0{,}675} = 0{,}822$$

$$\Omega_u = 0{,}822 \cdot \omega_0 = 0{,}822 \cdot 44{,}29\,\frac{1}{s} = 36{,}41\,\frac{1}{s}$$

$$\Omega_o = 1{,}405 \cdot \omega_0 = 1{,}405 \cdot 44{,}29\,\frac{1}{s} = 62{,}23\,\frac{1}{s}$$

zu 4) $\Omega = \dfrac{2\pi \cdot n}{60} = \dfrac{2\pi \cdot 650\,\mathrm{min}^{-1}}{60\,\frac{s}{\mathrm{min}}} = 68{,}07\,\frac{1}{s}$

$$\eta = \frac{\Omega}{\omega_0} = \frac{68{,}07\,\frac{1}{s}}{44{,}29\,\frac{1}{s}} = 1{,}537$$

$$\hat{w} = \frac{1{,}537^2}{\sqrt{(1-1{,}537^2)^2 + 4 \cdot 0{,}0562^2 \cdot 1{,}537^2}} \cdot \frac{0{,}3\,\mathrm{kg \cdot m}}{100\,\mathrm{kg}}$$

$$\hat{w} = 0{,}00516\,\mathrm{m} = 5{,}16\,\mathrm{mm}$$

1) $(\underline{C} - \omega^2 \cdot \underline{M}) \cdot \underline{\hat{x}} = \underline{0}$

$$\det(\underline{C} - \omega^2 \cdot \underline{M}) = 0$$

$$\det \begin{bmatrix} 5\cdot c - \omega^2 \cdot m_1 & -2\cdot c \\ -2\cdot c & 3\cdot c - \omega^2 \cdot m_2 \end{bmatrix} = 0$$

$$(5\cdot c - \omega^2 \cdot m_1)\cdot(3\cdot c - \omega^2 \cdot m_2) - 4\cdot c^2 = 0$$

$$15c^2 - \omega^2 \cdot 5c \cdot m_2 - \omega^2 \cdot 3c \cdot m_1 + \omega^4 \cdot m_1 \cdot m_2 - 4c^2 = 0$$

$$m_1 \cdot m_2 \cdot \omega^4 - (3c \cdot m_1 + 5c \cdot m_2)\cdot \omega^2 + 11c^2 = 0$$

$$50\,\cancel{kg^2} \cdot \omega^4 - \left(30000\,\frac{\cancel{kg^2}}{s^2} + 25000\,\frac{\cancel{kg^2}}{s^2}\right)\cdot \omega^2 + 11\cdot 10^6\,\frac{\cancel{kg^2}}{s^4} = 0$$

$$\omega^4 - 1100 \cdot \omega^2 + 220000 = 0$$

$$\xi^2 - 1100 \cdot \xi + 220000 = 0$$

$$\xi_{1/2} = 550 \mp \sqrt{550^2 - 220000}$$

$$\xi_1 = 262{,}77\,\frac{1}{s^2}\ ; \quad \omega_1 = 16{,}21\,\frac{1}{s}$$

$$\xi_2 = 837{,}23\,\frac{1}{s^2}\ ; \quad \omega_2 = 28{,}93\,\frac{1}{s}$$

$$\underline{\xi} = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} = \begin{bmatrix} 262{,}77\,\frac{1}{s^2} & 0 \\ 0 & 837{,}23\,\frac{1}{s^2} \end{bmatrix}$$

2) 1. Eigenvektor

$$\begin{bmatrix} 2372,3 & -2000 \\ -2000 & 1686,15 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} = \underline{0}$$

$$2372,3 \cdot \hat{x}_{11} - 2000 \cdot \hat{x}_{12} = 0 \Rightarrow \hat{x}_{11} = 0,843 \cdot \hat{x}_{12}$$

$$-2000 \cdot \hat{x}_{11} + 1686,15 \cdot \hat{x}_{12} = 0 \Rightarrow \hat{x}_{11} = 0,843 \cdot \hat{x}_{12}$$

$$\underline{\hat{x}}_1 = \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} = \begin{bmatrix} 0,843 \\ 1 \end{bmatrix}$$

2. Eigenvektor

$$\begin{bmatrix} -3372,3 & -2000 \\ -2000 & -1186,15 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} = \underline{0}$$

$$-3372,3 \cdot \hat{x}_{21} - 2000 \cdot \hat{x}_{22} = 0 \Rightarrow \hat{x}_{21} = -0,593 \cdot \hat{x}_{22}$$

$$-2000 \cdot \hat{x}_{21} - 1186,15 \cdot \hat{x}_{22} = 0 \Rightarrow \hat{x}_{21} = -0,593 \cdot \hat{x}_{22}$$

$$\underline{\hat{x}}_2 = \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} = \begin{bmatrix} -0,593 \\ 1 \end{bmatrix}$$

$$\underline{\hat{x}}_1^T \cdot \underline{M} \cdot \underline{\hat{x}}_1 = m_1^*$$

$$\begin{array}{ccc} & \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} & \begin{bmatrix} 0,843 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0,843 & 1 \end{bmatrix} & \begin{bmatrix} 8,43 & 5 \end{bmatrix} & \begin{bmatrix} 12,11 \end{bmatrix} = m_1^* \end{array}$$

$$\begin{array}{ccc} & \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} & \begin{bmatrix} -0,593 \\ 1 \end{bmatrix} \\ \begin{bmatrix} -0,593 & 1 \end{bmatrix} & \begin{bmatrix} -5,93 & 5 \end{bmatrix} & \begin{bmatrix} 8,52 \end{bmatrix} = m_2^* \end{array}$$