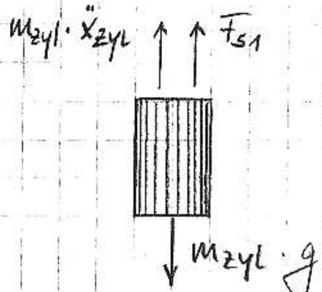
zu 1)

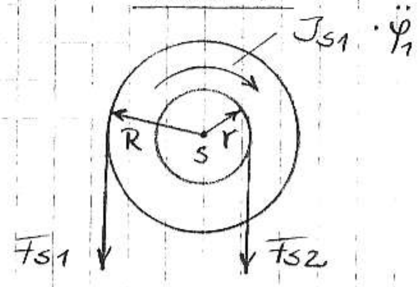
Zylinder



$$m_{zyl} \cdot \ddot{x}_{zyl} + F_{S1} = m_{zyl} \cdot g$$

$$F_{S1} = m_{zyl} \cdot (g - \ddot{x}_{zyl}) \quad (1)$$

Rolle 1

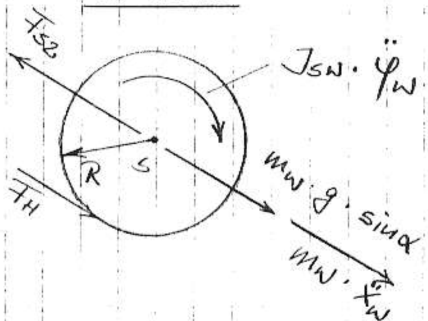


$\sum M_{iS} = 0$

$$F_{S1} \cdot R - F_{S2} \cdot r - J_{S1} \cdot \ddot{\varphi}_1 = 0$$

$$F_{S1} = F_{S2} \cdot \frac{r}{R} + J_{S1} \cdot \frac{\ddot{\varphi}_1}{R} \quad (2)$$

Walze



$\sum F_{it} = 0$

$$F_{S2} - m_W \cdot g \cdot \sin\alpha - m_W \cdot \ddot{x}_W - F_H = 0$$

$$F_{S2} = F_H + m_W \cdot g \cdot \sin\alpha + m_W \cdot \ddot{x}_W \quad (3)$$

$\sum M_{iS} = 0$

$$F_H \cdot R = J_{SW} \cdot \ddot{\varphi}_W$$

$$F_H = J_{SW} \cdot \frac{\ddot{\varphi}_W}{R} \quad (4)$$

Kinematik

$$\varphi_1 = \frac{x_{zyl}}{R} = \frac{x_W}{r} \; ; \quad x_W = x_{zyl} \cdot \frac{r}{R} \; ;$$

$$\varphi_W = \frac{x_W}{R} = x_{zyl} \cdot \frac{r}{R^2} \quad (5)$$

$$(1) = (2) \quad F_{S2} \cdot \frac{r}{R} + J_{S1} \cdot \frac{\ddot{\varphi}_1}{R} = m_{zyl} \cdot (g - \ddot{x}_{zyl}) \quad (6)$$

$$(4) \text{ in } (3) \quad F_{S2} = J_{SW} \cdot \frac{\ddot{\varphi}_W}{R} + m_W \cdot g \cdot \sin\alpha + m_W \cdot \ddot{x}_W$$

in ⑥ $\left(J_{SW} \cdot \frac{\ddot{\varphi}_W}{R} + m_W \cdot \underset{\smile}{g} \cdot \sin\alpha + m_W \cdot \ddot{x}_W \right) \cdot \frac{r}{R} + J_{S1} \cdot \frac{\ddot{\varphi}_1}{R}$ 1/2

$= m_{zyl} \cdot \left(\underset{\smile}{g} - \underset{\smile}{\ddot{x}_{zyl}} \right)$

mit ⑤ $\left(J_{SW} \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r}{R^3} + m_W \cdot \underset{\smile}{g} \cdot \sin\alpha + m_W \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r}{R} \right) \cdot \frac{r}{R}$

$+ J_{S1} \cdot \frac{\underset{\smile}{\ddot{x}_{zyl}}}{R^2} = m_{zyl} \cdot \underset{\smile}{g} - m_{zyl} \cdot \underset{\smile}{\ddot{x}_{zyl}}$

$\left(\frac{1}{2} \cdot m_W \cdot R^2 \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r}{R^3} + m_W \cdot \underset{\smile}{g} \cdot \sin\alpha + m_W \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r}{R} \right) \cdot \frac{r}{R}$

$+ \frac{1}{2} \cdot m_{R1} \cdot R^2 \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{1}{R^2} = m_{zyl} \cdot \underset{\smile}{g} - m_{zyl} \cdot \underset{\smile}{\ddot{x}_{zyl}}$

$\frac{1}{2} \cdot m_W \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r^2}{R^2} + m_W \cdot \underset{\smile}{g} \cdot \frac{r}{R} \cdot \sin\alpha + m_W \cdot \underset{\smile}{\ddot{x}_{zyl}} \cdot \frac{r^2}{R^2}$

$+ \frac{1}{2} \cdot m_{R1} \cdot \underset{\smile}{\ddot{x}_{zyl}} = m_{zyl} \cdot \underset{\smile}{g} - m_{zyl} \cdot \underset{\smile}{\ddot{x}_{zyl}}$

$\left(\frac{1}{2} \cdot m_W \cdot \frac{r^2}{R^2} + m_W \cdot \frac{r^2}{R^2} + \frac{1}{2} \cdot m_{R1} + m_{zyl} \right) \cdot \underset{\smile}{\ddot{x}_{zyl}}$ ⑦

$= m_{zyl} \cdot \underset{\smile}{g} - m_W \cdot \underset{\smile}{g} \cdot \frac{r}{R} \cdot \sin\alpha$

$\left(10\,\underset{\smile}{kg} + 20\,\underset{\smile}{kg} + 20\,\underset{\smile}{kg} + 50\,\underset{\smile}{kg} \right) \cdot \underset{\smile}{\ddot{x}_{zyl}} = \left(50\,\underset{\smile}{kg} - 20\,\underset{\smile}{kg} \right) \cdot 9{,}81\,\frac{m}{s^2}$

$\underset{\smile}{\ddot{x}_{zyl}} = \frac{294{,}3\ \cancel{kg} \cdot \frac{m}{s^2}}{100\ \cancel{kg}} = 2{,}94\ \frac{m}{s^2}$

$\ddot{x}_W = \frac{1}{2}\, \underset{\smile}{\ddot{x}_{zyl}} = 1{,}47\ \frac{m}{s^2}$

zu 2) $\dot{x}_W = \sqrt{2 \cdot \ddot{x}_W \cdot x_W} = \sqrt{2 \cdot 1{,}47 \frac{m}{s^2} \cdot 2m} = 2{,}42\ \frac{m}{s}$

zu 3) $F_H = J_{SW} \cdot \frac{\ddot{\varphi}_W}{R} = \frac{1}{2} \cdot m_W \cdot R^2 \cdot \frac{\ddot{x}_W}{R^2} = \frac{1}{2} \cdot 80\,\underset{\smile}{kg} \cdot 1{,}47\,\frac{m}{s^2}$

$F_H = 58{,}8\ N$

zu 4) aus Gl. ⑦ mit $\ddot{x}_{zyl} = 0$

$$m_{zyl} \cdot g = m_W \cdot g \cdot \frac{r}{R} \cdot \sin\alpha$$

$$m_W = m_{zyl} \cdot \frac{R}{r \cdot \sin\alpha}$$

$$m_W = 50\,kg \cdot \frac{0{,}20\,m}{0{,}10\,m \cdot 0{,}5} = 200\,kg$$

Aufgabe 2

zu 1) $\omega_0 = \sqrt{\frac{c_{ges}}{m}} = \sqrt{\frac{6 \cdot 10^5\ N/m}{600\ kg}} = 31{,}62\ \frac{1}{s}$

$$\vartheta = \frac{d}{2 \cdot m \cdot \omega_0} = \frac{2000\ kg \cdot s^{-1}}{2 \cdot 600\ kg \cdot 31{,}62\ \frac{1}{s}} = 0{,}0527$$

zu 2)

$$b_K = \frac{2}{m} \cdot \sum_{i=0}^{m-1} u(t_i) \cdot \sin\left(\frac{2 \cdot k \cdot i \cdot \pi}{m}\right)$$

i	t_i [s]	$u(t_i)$ [mm]	$\sin\left(\frac{2 \cdot i \cdot \pi}{m}\right)$	$\sin\left(\frac{4 \cdot i \cdot \pi}{m}\right)$	$u(t_i) \cdot \sin\left(\frac{2 \cdot i \cdot \pi}{m}\right)$ [mm]	$u(t_i) \cdot \sin\left(\frac{4 \cdot i \cdot \pi}{m}\right)$
0	0	0	0	0	0	0
1	0,06	2,5981	0,8660	0,8660	2,25	2,25
2	0,12	0,8660	0,8660	−0,8660	0,75	−0,75
3	0,18	0	0	0	0	0
4	0,24	−0,8660	−0,8660	0,8660	0,75	−0,75
5	0,30	−2,5981	−0,8660	−0,8660	2,25	2,25
					6	3

$$b_1 = \frac{2}{6} \cdot 6 = 2\ mm$$

$$b_2 = \frac{2}{6} \cdot 3 = 1\ mm$$

$$\Omega = \frac{2\pi}{T} = \frac{2\pi}{0{,}36s} = 17{,}453\ \frac{1}{s}$$

$$u(t) = 2\ mm \cdot \sin\left(17{,}453\ \frac{1}{s} \cdot t\right) + 1\ mm \cdot \sin\left(34{,}907\ \frac{1}{s} \cdot t\right)$$

zu 3)

$$\eta_1 = \frac{17{,}453\ \frac{1}{s}}{31{,}62\ \frac{1}{s}} = 0{,}552$$

$$\hat{x}_1 = \frac{\sqrt{1+4\cdot 0{,}0527^2\cdot 0{,}552^2}\cdot 2\,mm}{\sqrt{(1-0{,}552^2)^2+4\cdot 0{,}0527^2\cdot 0{,}552^2}} = \frac{2{,}0033\,mm}{0{,}6977}$$

$$\hat{x}_1 = 2{,}87\,mm$$

$$\varphi_1 = -\arctan\left(\frac{2\cdot 0{,}0527\cdot 0{,}552^3}{1-0{,}552^2+4\cdot 0{,}0527^2\cdot 0{,}552^2}\right)$$

$$\varphi_1 = -0{,}025$$

$$\eta_2 = 2\cdot 0{,}552 = 1{,}104$$

$$\hat{x}_2 = \frac{\sqrt{1+4\cdot 0{,}0527^2\cdot 1{,}104^2}\cdot 1\,mm}{\sqrt{(1-1{,}104^2)^2+4\cdot 0{,}0527^2\cdot 1{,}104^2}} = \frac{1{,}0067\,mm}{0{,}2478}$$

$$\hat{x}_2 = 4{,}06\,mm$$

$$\varphi_2 = -\arctan\left(\frac{2\cdot 0{,}0527\cdot 1{,}104^3}{1-1{,}104^2+4\cdot 0{,}0527^2\cdot 1{,}104^2}\right) = -\arctan(-0{,}6909)$$

$$\varphi_2 = -2{,}537$$

$$x(t) = 2{,}87\,mm\cdot\sin\left(17{,}453\,\tfrac{1}{s}\cdot t - 0{,}025\right)$$
$$+\ 4{,}06\,mm\cdot\sin\left(34{,}907\,\tfrac{1}{s}\cdot t - 2{,}537\right)$$

Aufgabe 3

zu 1)

$$\det \begin{bmatrix} 24000 - \omega^2 \cdot 150 & -16000 \\ -24000 & 24000 - \omega^2 \cdot 60 \end{bmatrix} = 0$$

$$(24000 - \omega^2 \cdot 150) \cdot (24000 - \omega^2 \cdot 60) - (24000) \cdot (-16000) = 0$$

$$576 \cdot 10^6 - 1440000 \cdot \omega^2 - 3600000 \cdot \omega^2 + 9000 \cdot \omega^4$$
$$- 384 \cdot 10^6 = 0$$

$$9000 \cdot \omega^4 - 5040000 \cdot \omega^2 + 192000000 = 0$$

$$\omega^4 - 560 \cdot \omega^2 + 21333,\bar{3} = 0$$

$$\omega_1^2 = 280 - \sqrt{78400 - 21333,\bar{3}} = 41,11 \tfrac{1}{s^2}$$

$$\omega_2^2 = 280 + \sqrt{78400 - 21333,\bar{3}} = 518,89 \tfrac{1}{s^2}$$

$$\omega_1 = 6,41 \tfrac{1}{s} \; ; \; \omega_2 = 22,78 \tfrac{1}{s}$$

zu 2) 1. Eigenvektor

$$\begin{bmatrix} 17833,5 & -16000 \\ -24000 & 21533,4 \end{bmatrix} \cdot \begin{bmatrix} \hat{\varphi}_{11} \\ \hat{\varphi}_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

obere Gleichung: $17833,5 \cdot \hat{\varphi}_{11} - 16000 \cdot \hat{\varphi}_{12} = 0 \Rightarrow \hat{\varphi}_{11} = 0,897 \cdot \hat{\varphi}_{12}$

untere Gleichung: $-24000 \cdot \hat{\varphi}_{11} + 21533,4 \cdot \hat{\varphi}_{12} = 0 \Rightarrow \hat{\varphi}_{11} = 0,897 \cdot \hat{\varphi}_{12}$

gewählt: $\hat{\varphi}_{12} = 0,1 \Rightarrow \hat{\varphi}_{11} = 0,0897$

$$\underline{\hat{\varphi}_1} = \begin{bmatrix} 0{,}0897 \\ 0{,}1 \end{bmatrix}$$

2. Eigenvektor

$$\begin{bmatrix} -53833{,}5 & -16000 \\ -24000 & -7133{,}4 \end{bmatrix} \cdot \begin{bmatrix} \hat{\varphi}_{21} \\ \hat{\varphi}_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

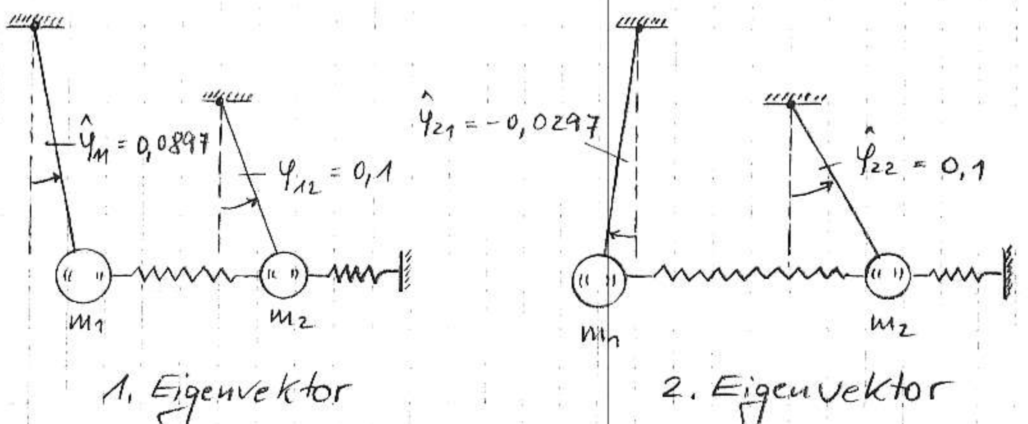
obere Gleichung: $-53833{,}5 \cdot \hat{\varphi}_{21} - 16000 \cdot \hat{\varphi}_{22} = 0 \Rightarrow \hat{\varphi}_{21} = -0{,}297 \cdot \hat{\varphi}_{22}$

untere Gleichung: $-24000 \cdot \hat{\varphi}_{21} - 7133{,}4 \cdot \hat{\varphi}_{22} = 0 \Rightarrow \hat{\varphi}_{21} = -0{,}297 \cdot \hat{\varphi}_{22}$

gewählt: $\hat{\varphi}_{22} = 0{,}1 \Rightarrow \hat{\varphi}_{21} = -0{,}0297$

$$\underline{\hat{\varphi}_2} = \begin{bmatrix} -0{,}0297 \\ 0{,}1 \end{bmatrix}$$

zu 3)



zu 4)

$$\underline{\Phi} = \begin{bmatrix} 0{,}0897 & -0{,}0297 \\ 0{,}1 & 0{,}1 \end{bmatrix}; \quad \underline{S} = \begin{bmatrix} 41{,}11 \frac{1}{s^2} & 0 \\ 0 & 518{,}89 \frac{1}{s^2} \end{bmatrix}$$