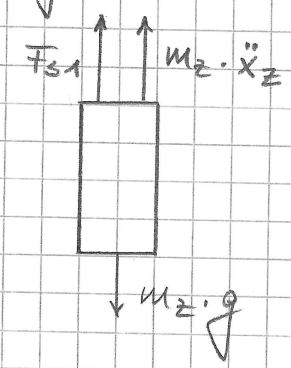
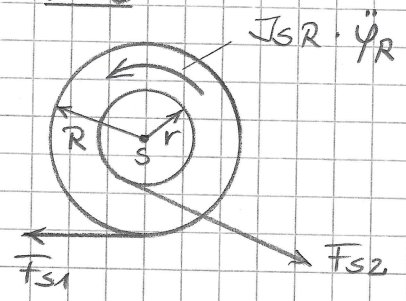
Zylinder



$$F_{S1} = m_Z \cdot (g - \ddot{x}_Z) \quad ①$$

Rolle

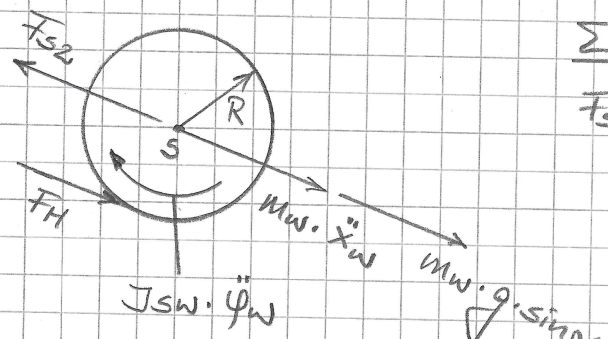


$$\sum M_{iS} = 0$$

$$F_{S2} \cdot r - F_{S1} \cdot R + J_{SR} \cdot \ddot{\varphi}_R = 0$$

$$F_{S1} = F_{S2} \cdot \frac{r}{R} + \frac{J_{SR}}{R} \cdot \ddot{\varphi}_R \quad ②$$

Walze



$$\sum F_{it} = 0$$

$$F_{S2} = m_W \cdot \ddot{x}_W + m_W \cdot g \cdot \sin\alpha + F_H \quad ③$$

$$\sum M_{iS} = 0$$

$$F_H \cdot R - J_{SW} \cdot \ddot{\varphi}_W = 0$$

$$F_H = \frac{J_{SW}}{R} \cdot \ddot{\varphi}_W \quad ④$$

Kinematik

$$\ddot{\varphi}_W = \frac{\ddot{x}_W}{R} = \ddot{x}_Z \cdot \frac{r}{R^2} \; ; \; \ddot{\varphi}_R = \frac{\ddot{x}_Z}{R} \; ; \; \ddot{x}_W = \ddot{x}_Z \cdot \frac{r}{R}$$

① = ②: $m_Z \cdot g - m_Z \cdot \ddot{x}_Z = F_{S2} \cdot \frac{r}{R} + \frac{J_{SR}}{R} \cdot \ddot{\varphi}_R$

④ in ③: $F_{S2} = m_W \cdot \ddot{x}_W + m_W \cdot g \cdot \sin\alpha + \frac{J_{SW}}{R} \cdot \ddot{\varphi}_W$

$$m_Z \cdot g - m_Z \cdot \ddot{x}_Z = m_W \cdot \frac{r}{R} \cdot \ddot{x}_W + m_W \cdot g \cdot \frac{r}{R} \cdot \sin\alpha + \frac{J_{SW}}{R^2} \cdot r \cdot \ddot{\varphi}_W + \frac{J_{SR}}{R} \cdot \ddot{\varphi}_R$$

$$m_z \cdot g - m_z \cdot \ddot{x}_z = m_w \cdot \frac{r^2}{R^2} \cdot \ddot{x}_z + m_w \cdot g \cdot \frac{r}{R} \cdot \sin\alpha$$

$$+ \frac{1}{2} \cdot m_w \cdot \frac{r^2}{R^2} \cdot \ddot{x}_z + \frac{1}{2} \cdot m_R \cdot \ddot{x}_z$$

$$\left(m_w \cdot \frac{r^2}{R^2} + \frac{1}{2} \cdot m_w \cdot \frac{r^2}{R^2} + \frac{1}{2} \cdot m_R + m_z \right) \cdot \ddot{x}_z =$$

$$m_z \cdot g - m_w \cdot g \cdot \frac{r}{R} \cdot \sin\alpha$$

$$\ddot{x}_z = \frac{m_z \cdot g - m_w \cdot g \cdot \frac{r}{R} \cdot \sin\alpha}{\frac{3}{2} \cdot m_w \cdot \frac{r^2}{R^2} + \frac{1}{2} \cdot m_R + m_z}$$

$$\ddot{x}_z = \frac{60\,kg \cdot 9{,}81\,\frac{m}{s^2} - 50\,kg \cdot 9{,}81\,\frac{m}{s^2} \cdot \frac{1}{2} \cdot \sin 30°}{\frac{3}{2} \cdot 50\,kg \cdot \frac{1}{4} + \frac{1}{2} \cdot 20\,kg + 60\,kg}$$

$$\ddot{x}_z = \frac{465{,}975\,\frac{kg \cdot m}{s^2}}{88{,}75\,kg} = 5{,}25\,\frac{m}{s^2}$$

$$\ddot{x}_w = 2{,}63\,\frac{m}{s^2}$$

zu 2) $$x_z = 2 \cdot \pi \cdot R \cdot \frac{R}{r} = 2 \cdot \pi \cdot \frac{R^2}{r}$$

$$x_z = 2 \cdot \pi \cdot \frac{(0{,}2\,m)^2}{0{,}1\,m} = 2{,}51\,m$$

$$\dot{x}_z = \sqrt{2 \cdot \ddot{x}_z \cdot x_z} = \sqrt{2 \cdot 5{,}25\,\frac{m}{s^2} \cdot 2{,}51\,m}$$

$$\dot{x}_z = 5{,}13\,\frac{m}{s}$$

zu 3) $F_H = \frac{J_{SW}}{R} \cdot \ddot{x}_2 \cdot \frac{r}{R^2}$

$$F_H = \frac{1}{2} \cdot m_W \cdot \frac{r}{R} \cdot \ddot{x}_2 = \frac{1}{2} \cdot 50\,kg \cdot \frac{1}{2} \cdot 5{,}25\,\frac{m}{s^2}$$

$$F_H = 65{,}63\,N$$

zu 4) $\mu \cdot F_N \geq F_H$

$$\mu \geq \frac{F_H}{F_N} = \frac{F_H}{m_W \cdot g \cdot \cos\alpha}$$

$$\mu \geq \frac{65{,}63\,N}{50\,kg \cdot 9{,}81\,\frac{m}{s^2} \cdot \cos 30°}$$

$$\mu \geq 0{,}155$$

$y(t)$ ist eine gerade Funktion $\Rightarrow b_k = 0$

Fourierkoeffizient a_0

$$a_0 = \frac{2}{T} \cdot \int_0^{\frac{T}{2}} y(t) \cdot dt = \frac{2}{T} \cdot \int_0^{\frac{T}{6}} 2 \cdot h \cdot dt + \frac{2}{T} \cdot \int_{\frac{T}{6}}^{\frac{T}{3}} h \cdot dt + 0$$

$$a_0 = \frac{4 \cdot h}{T} \cdot \frac{T}{6} + \frac{2 \cdot h}{T} \cdot \left(\frac{T}{3} - \frac{T}{6} \right) = h$$

Fourierkoeffizienten a_k

$$a_k = \frac{4}{T} \cdot \int_0^{\frac{T}{2}} y(t) \cdot \cos(k \cdot \omega \cdot t) \cdot dt$$

$$a_k = \frac{4}{T} \cdot \int_0^{\frac{T}{6}} 2 \cdot h \cdot \cos(k \cdot \omega \cdot t) \cdot dt + \frac{4}{T} \cdot \int_{\frac{T}{6}}^{\frac{T}{3}} h \cdot \cos(k \cdot \omega \cdot t) \cdot dt$$

$$a_k = \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \sin(k \cdot \omega \cdot t) \Big|_0^{\frac{T}{6}} + \frac{4 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \sin(k \cdot \omega \cdot t) \Big|_{\frac{T}{6}}^{\frac{T}{3}}$$

$$a_k = \frac{8 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \sin\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{6} \right)$$

$$\frac{4 \cdot h}{T} \cdot \frac{1}{k \cdot \omega} \cdot \left[\sin\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{3} \right) - \sin\left(k \cdot \frac{2\pi}{T} \cdot \frac{T}{6} \right) \right]$$

$$a_k = \frac{4 \cdot h}{k \cdot \pi} \cdot \sin\left(k \cdot \frac{\pi}{3} \right) + \frac{2 \cdot h}{k \cdot \pi} \cdot \sin\left(\frac{2 \cdot k \cdot \pi}{3} \right)$$

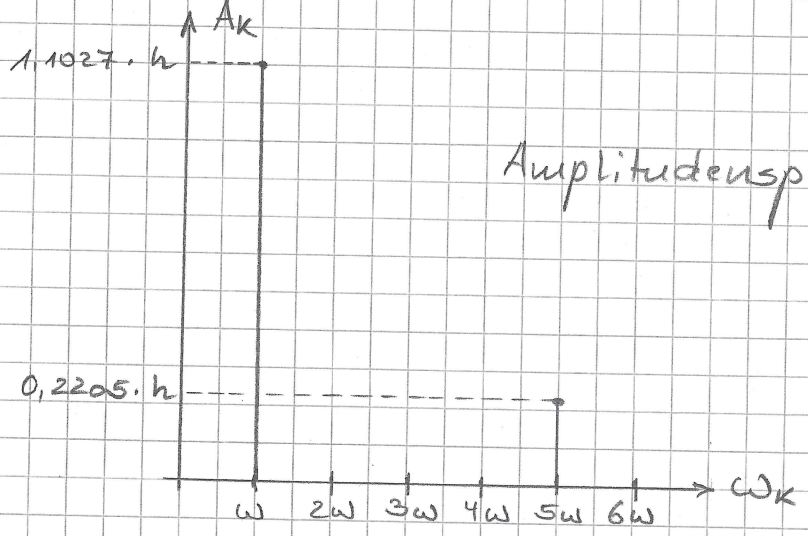
$$- \frac{2 \cdot h}{k \cdot \pi} \cdot \sin\left(k \cdot \frac{\pi}{3} \right)$$

$$a_k = \frac{2 \cdot h}{k \cdot \pi} \cdot \left[\sin\left(k \cdot \frac{\pi}{3}\right) + \sin\left(\frac{2 \cdot k \cdot \pi}{3}\right) \right]$$

k	$\sin\left(\frac{k \cdot \pi}{3}\right)$	$\sin\left(\frac{2 \cdot k \cdot \pi}{3}\right)$	a_k
1	0,8660	0,8660	$1{,}1027 \cdot h$
2	0,8660	−0,8660	0
3	0	0	0
4	−0,8660	0,8660	0
5	−0,8660	−0,8660	$-0{,}2205 \cdot h$
6	0	0	0

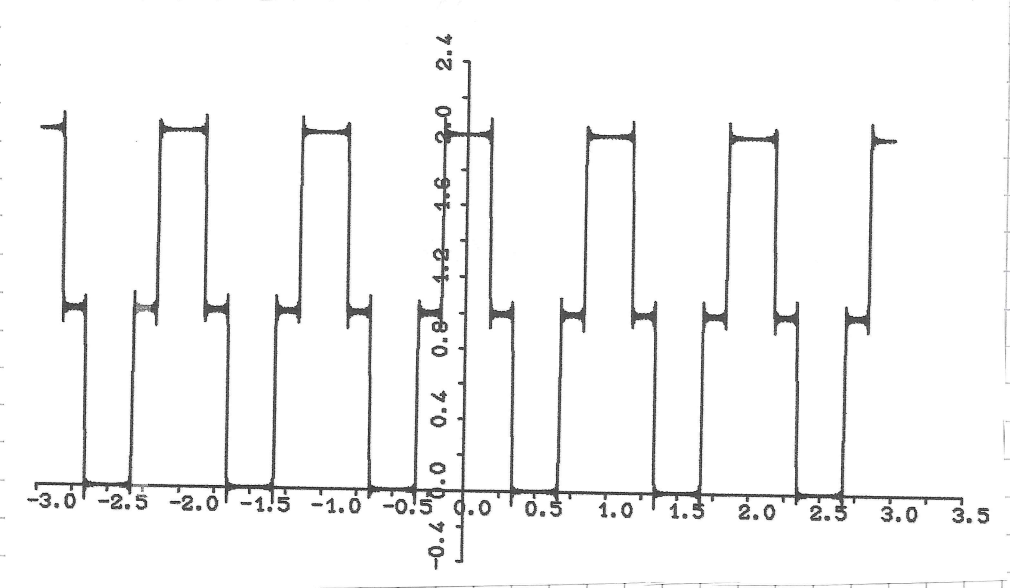
$$y(t) = h + h \cdot \left[1{,}1027 \cdot \cos(\omega \cdot t) - 0{,}2205 \cdot \cos(5 \cdot \omega \cdot t) + \dots \right]$$

$$A_k = \sqrt{a_k^2 + b_k^2} \quad \text{mit } b_k = 0 \Rightarrow A_k = \sqrt{a_k^2} = |a_k|$$



Amplitudenspektrum

Auswertung: $h = 1{,}0$; $T = 1{,}0\,s$; 100 Harmonische



zu 1)

$$c = \frac{12 \cdot 2{,}1 \cdot 10^5 \frac{N}{mm^2} \cdot 843 \cdot 10^4 mm^4}{(2000mm)^2 \cdot (4 \cdot 2000mm + 3000mm)}$$

$$c = 312{,}406 \frac{N}{mm} = 312406 \frac{N}{m}$$

$$\omega_0 = \sqrt{\frac{312406 \frac{N}{m}}{200kg}} = 39{,}52 \frac{1}{s}$$

zu 2) a) $$\Lambda = \frac{1}{4} \cdot \ln\left(\frac{1cm}{0{,}4cm}\right) = 0{,}2291$$

$$D = \frac{\Lambda}{\sqrt{4 \cdot \pi^2 + \Lambda^2}} = \frac{0{,}2291}{\sqrt{4 \cdot \pi^2 + 0{,}2291^2}} = 0{,}036$$

b) $$\omega_d = 39{,}52 \frac{1}{s} \cdot \sqrt{1 - 0{,}036^2} = 39{,}49 \frac{1}{s}$$

$$\delta = D \cdot \omega_0 = 0{,}036 \cdot 39{,}52 \frac{1}{s} = 1{,}423 \frac{1}{s}$$

$$C_1 = W_0 = 1cm$$

$$C_2 = \frac{\dot{W}_0 + \delta \cdot W_0}{\omega_d} = \frac{1{,}423 \frac{1}{s} \cdot 1cm}{39{,}49 \frac{1}{s}} = 0{,}036\,cm$$

$$W(t) = e^{-1{,}423 \frac{1}{s} \cdot t} \cdot \left[1cm \cdot \cos\left(39{,}49 \frac{1}{s} \cdot t\right) + 0{,}036\,cm \cdot \sin\left(39{,}49 \frac{1}{s} \cdot t\right)\right]$$

zu 3) $$\Omega = \frac{2\pi \cdot 300 min^{-1}}{60 \frac{s}{min}} = 31{,}42 \frac{1}{s}$$

$$\eta_1 = \frac{\Omega}{\omega_0} = \frac{31{,}42 \frac{1}{s}}{39{,}52 \frac{1}{s}} = 0{,}795$$

$$\hat{w}_1 = \frac{1}{\sqrt{(1-0{,}795^2)^2 + 4 \cdot 0{,}036^2 \cdot 0{,}795^2}} \cdot \frac{1000\,N}{312{,}406\,\frac{N}{mm}}$$

$$\hat{w}_1 = 8{,}60\,mm$$

$$\varphi_1 = -\arctan\left(\frac{2 \cdot 0{,}036 \cdot 0{,}795}{1 - 0{,}795^2}\right) = -0{,}1543$$

$$\eta_2 = \frac{2 \cdot \Omega}{\omega_0} = \frac{2 \cdot 31{,}42\,\frac{1}{s}}{39{,}52\,\frac{1}{s}} = 1{,}59$$

$$\hat{w}_2 = \frac{1}{\sqrt{(1-1{,}59^2)^2 + 4 \cdot 0{,}036^2 \cdot 1{,}59^2}} \cdot \frac{600\,N}{312{,}406\,\frac{N}{mm}}$$

$$\hat{w}_2 = 1{,}25\,mm$$

$$\varphi_2 = -\arctan\left(\frac{2 \cdot 0{,}036 \cdot 1{,}59}{1 - 1{,}59^2}\right) = -3{,}067$$

$$w(t) = 8{,}60\,mm \cdot \sin\left(31{,}42\,\frac{1}{s} \cdot t - 0{,}1543\right) + 1{,}25\,mm \cdot \sin\left(62{,}84\,\frac{1}{s} \cdot t - 3{,}067\right)$$

zu 4)

$$\Omega = \omega_0 \Rightarrow n_1 = \frac{\omega_0}{2\pi} \cdot 60 = \frac{39{,}52\,\frac{1}{s}}{2\pi} \cdot 60\,\frac{s}{min}$$

$$n_1 = 377{,}4\,min^{-1}$$

$$2 \cdot \Omega = \omega_0 \Rightarrow n_2 = \frac{\frac{\omega_0}{2}}{2\pi} \cdot 60 = 188{,}7\,min^{-1}$$